

A NONLINEAR PROGRAMMING APPLICATION FOR OPTIMIZATION
OF ELECTRICITY CAPACITY
EXPANSION AND EMISSION PLANNING PROBLEMS

By

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TABLE OF CONTENTS

	<u>Page</u>
ACKNOWLEDGMENTS	ii
ABSTRACT	vi
CHAPTERS	
1 INTRODUCTION	1
Purpose	1
Problem	2
Title IV of Clean Air Act Amendments	2
Phase I Sulfur Dioxide Reduction	3
Phase II - Sulfur Dioxide Reduction	4
The Emission Allowance System	4
Allowances Trading	5
Compliance Options	6
Scope	10
2 LITERATURE REVIEW	12
Emission Reduction Evaluation Models	12
Linear, Nonlinear and	
Dynamic Programming Models	13
Individual SO ₂ Reduction Models	17
Heuristic Models	18
Summary	21
3 CAPACITY EXPANSION AND EMISSION CONTROL PROBLEM	
FORMULATION	24
Emission Capacity Control Problem	
(ECCP) for Electric Utility	24
Probabilistic Production Costing	25
Merit Order	29

	The Deterministic Case (No Failures Considered)	29
	The Non-deterministic Case (Failures Considered)	30
	The Formulation of Capacity Controls Problem	39
4	SOLUTION PROCEDURE	46
	Deriving the Subproblem	48
	The Dual Subproblem	55
	The Master Problem	63
	Conclusion	70
5	PRODUCTION PROBABILISTIC COSTING FUNCTION AND ITS DERIVATIVES	71
	The Cumulant Method	71
	The Formula of the Cumulants Method	73
	Advantages of the Cumulants Method	81
	The Derivatives of The Objective Function	82
6	COMPUTATIONAL RESULTS FOR EXAMPLE UTILITY SYSTEM	90
	Outline of the Solution Procedure	90
	Gainesville Regional Utilities System	92
	Load Data	93
	Retrofit Data	94
	Expansion Data	97
	ECCP Formulation	97
	The Least Costs for GRU	98
	Sensitivity Analysis	101
	Emission Allowance Price Escalation	103
	Demand Growth Forecast	104
	Fuel Price Escalation	107
	Gas price analysis	107
	Coal price analysis	108
	Reserve Capacity Requirements	110
	Scaled Down Electric Power Research Institute (EPRI) System E	111
	Third Example (Huang and Hobbs Example)	116
	Description of the System	116
	The Least Cost Plan	119

7	SUMMARY AND CONCLUSION	122
	Summary and Contribution	122
	Future Research	126
	Hydroelectric Power Plants	127
	Fixed-Size Plants and Their Costs to Build	128
	No Retrofit of an SO ₂ Option on New Plant	128
	Demand Uncertainty	128

APPENDICES

A	SPREADSHEET PROGRAM	130
B	ILLUSTRATION OF THE SOLUTION PROCEDURES APPLIED TO GRU	134
C	ILLUSTRATION OF THE SOLUTION PROCEDURES APPLIED TO EPRI SYSTEM E PROBLEM	140

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In November, 1990, the U.S. Congress passed the Clean Air Act Amendments of 1990 (CAAA-90) touching on virtually every aspect of existing air pollution law. Title IV (Acid Deposition Control) of CAAA-90 forms the centerpiece of the amendments affecting utilities. It is designed to specifically address sulfur dioxide (SO_2) emission levels by electric utilities. Because of the CAAA-90, electric utilities will be able to choose from a numerous wide range of SO_2 emissions control options. A critical decision that faces electric utilities is how to construct a comprehensive emissions control strategy that addresses all the criteria and all the alternatives available in an effort to find the most effective and least expensive way to comply with the legislation. In this study we will construct a comprehensive optimization

model that can systematically examine all available compliance options. Many emission control models solve the problem of finding the optimal plan for a utility in a specific period. However, CAAA-90 designates two phases to achieve the mandated emission reduction. Further, investments are long lived; demand is growing; generation mix and fuel prices are uncertain; therefore the static solution may not be optimal for the dynamic system. To build a long term compliance plan, a multi-year nonlinear programming controls model has been considered. The objective function is the total cost (investment cost, operation cost and emission cost) of different years. Three test problems were run using data based on the characteristics of Gainesville Regional Utilities and a synthetic utility system, called Scaled Down EPRI System D.

CHAPTER 1

INTRODUCTION

Purpose

In November, 1990, the U.S. Congress passed the Clean Air Act Amendments of 1990 (hereafter, CAAA-90) touching on virtually every aspect of existing air pollution law. Title IV (Acid Deposition Control) of CAAA-90 forms the centerpiece of the amendments affecting utilities, and it is designed to specifically reduce sulfur dioxide (SO₂) emission levels by electric utilities. The CAAA-90 does not specify how each utility will meet its requirements, and electric utilities will be able to choose their own compliance strategies from a numerous range of SO₂ emissions control options. A critical decision that faces electric utilities is how to construct a comprehensive emissions control strategy that addresses all the criteria and all the alternatives available in an effort to find the most effective and least expensive way to comply with the legislation. Models to evaluate the cost of SO₂ emissions from electric power plants for a particular compliance strategy have been constructed (McKee, 1993; Temple, Baker & Sloane, 1990). In this study we will construct a multi-year comprehensive optimization model that can systematically examine all available compliance options. Many emission control models solve the problem of finding the optimal plan for a utility in a specific period. However, CAAA-90

designates two phases to achieve the mandated emission reduction. Further, investments are long lived; demand is growing, generation mix and fuel prices are uncertain; therefore, the static solution may not be optimal for the dynamic system. To build a long term compliance plan, a multi-year nonlinear optimization model is needed. The objective function is the total cost (investment cost, operation cost and the emission cost) of different years.

Problem

In this study we will attempt to address the following concerns as a result of CAAA-90: What strategy and what type of emission control equipment should a utility install on each generating unit within its system in order to comply with the CAAA-90 at the least cost given the uncertainty of the demand growth, the uncertain prices of allowances, and changing fuel prices. What type and capacity power plant should a utility build in order to meet the growing demand?

Title IV of Clean Air Act Amendments

Title IV has two primary goals. The first major goal is the reduction of overall emissions of SO₂ by 10 million tons per year from the 1980 levels, capping emissions at 8.9 million tons by the year 2000. This represents a 50% reduction since, currently, 20 million tons of SO₂ are emitted by the electric utilities. In the United States, 55% of all the SO₂ emissions come from coal-fired electric generation plants (Tabors, 1989). The second goal of Title IV is to encourage energy conservation through the use of renewable energy.

The legislation will result in the reduction of current levels of acid rain and will produce many benefits for both the health of the population and the environment. "Every part of the country finally will have the means to attain healthy air on a realistic schedule. And the risk from toxic air emissions will be cut by three-fourths. Overall health risks, including risks of cancer, respiratory disease, heart ailments, and reproductive disorders, will be reduced dramatically (Reilly, 1990, p.4)". Other benefits will include increased visibility in metropolitan areas and reduction in U.S. dependency on the import of foreign oil.

Title IV of the CAAA-90 introduces an innovative new system of "allowances" (An "allowance" is an authorization to emit one ton of SO₂ in a specific calendar year) designed to assist utilities in their efforts to deal with the cost of complying with the SO₂ emission standards in the legislation. This allowance system, which will be discussed in the following section, is a unique method for easing the cost of compliance with Title IV and spreading the costs among those utilities affected.

Phase I Sulfur Dioxide Reduction

The implementation of Title IV is divided into two phases. Phase I will go into effect on January 1, 1995, and targets electric generation plants emitting over 2.5 pounds of SO₂ per million BTU of primary fuel consumed. This phase affects 261 power plants in the United States and will require them to emit no more than 2.5 pounds of SO₂ per million BTU.

Phase II - Sulfur Dioxide Reduction

Phase II will begin on January 1, 2000. Allowable utility SO₂ emission rates at that time will be lowered from 2.5 pounds per million BTU to 1.2 pounds per million BTU. Plants currently emitting less than 1.2 pounds per million BTU can increase their emission rates by roughly 20% between now and the year 2000. After January 1, 2000, emissions are to be capped at 8.9 million tons per year.

The Emission Allowance System

An elaborate system of allowances forms the heart of the emission reduction requirements of Title IV of the CAAA-90. Basically, an allowance represents an authorization to emit one ton of SO₂ in a specific calendar year. The Environmental Protection Agency (EPA) issues SO₂ emission allowances to the affected fossil fuel-fired power generation units according to a set of formulas specified under the legislation. Utilities that succeed in reducing their emission rates to levels below their annual requirement can carry unused allowances forward to future years, but allowances may not be used prior to the year for which they are issued. Surplus allowances also may be traded or sold to utilities whose units have emission rates that exceed the required levels. In Phase I of the legislation, effective January 1, 1995, utility units subject to emissions limitations will be allocated allowances based on the following formula:

average fuel use between 1985-1987 * 2.5 lb SO₂/MMBTU

In Phase II of the legislation, effective January 1, 2000, SO₂ emission limitations will apply to all existing and new utility units. The emission limitations on allowance allocations will then be based on the following formula:

$$\text{average fuel use between 1985-1987} * 1.2 \text{ lb SO}_2/\text{MMBTU}$$

If a utility reduces its SO₂ emission rate below its permitted emission level, then it will accumulate whatever allowances it does not use. These allowances can be saved for future use, transferred to another plant owned by the same company, or sold to another utility. If additional allowances are purchased, the purchasing utility may then emit beyond its original permitted emission level. The goal is to reduce overall emissions in the most effective manner possible while also allowing for flexibility in spreading the cost more evenly among the affected units. Utilities emitting in excess of their allowances will be subject to a fine of \$2,000 per ton of the excess emissions.

Allowances Trading

Under the CAAA-90 legislation, allowance holders can trade allowances, but trades must be recorded with the EPA. However, reallocations of allowances among affected units within a utility system generally will not have to be recorded with the EPA. Allowances can be traded before the year in which they become effective, but the allowances cannot actually be used prior to that calendar year. The bill also

includes provisions requiring that EPA regulations ensure electric reliability.

Temporary fluctuations (increases and decreases) in emissions over the year will be allowed as long as the unit meets its applicable annual emission requirements.

Compliance Options

Utilities throughout the U.S. are concerned about the impact of the CAAA-90 in terms of cost of compliance. Currently, the United States spends more than three times as much on environmental protection as it did in 1972 (Reilly, 1990). As a result of the CAAA-90, this figure is expected to continue to rise for the next fifteen years. According to the EPA, the U.S. will spend more than 2.7 percent of its Gross National Product in environmental protection by the end of the century. Utilities must try to meet the legislative requirements in the most cost effective ways. They must examine all the options available to reduce their emission rates. Many SO₂ control options, including energy conservation, minimum emission dispatch, use of low sulfur fuel, flue gas desulfurization (FGD), use of clean coal technology, and trading of emission allowances.

1) Energy Conservation:

Electricity conservation is one approach to SO₂ emissions control that has gained widespread attention. Conservation involves increasing the efficiency of end-use equipment such as appliances, motors, air conditioning units, and lights, and increasing the thermal integrity of buildings without lowering levels of comfort or

economic activity. Conservation reduces the amount of coal and other fuels that must be burned in a power plant to provide the same level of energy services. Therefore, this will directly lower emissions of SO_2 . In addition, conservation often reduces the net cost of energy services. Utility programs to promote energy conservation by the end-user can supplement other technological methods (e.g., installation of a scrubber on an existing power plant) to reduce SO_2 emissions. By reducing the amount of coal that must be burnt, energy conservation would contribute greatly to reducing the emission of SO_2 .

2) Minimum Emissions Dispatching:

Minimum Emissions Dispatching (MED) is the operation of an electric generation system in which generating units that have higher rates of emissions (on a lb/KWh basis) are used to produce less power. Thus, MED would result in fewer emissions than if these units were dispatched on a strict least-cost basis (Delson, 1974; Heslin and Hobbs, 1989). Traditionally, utilities dispatch generation units so as to minimize operating costs, subject to reliability constraints and other operating restrictions. Emissions dispatching adds to the dispatch problem a second objective--the lowering of emissions. The result is that cleaner units would generate more power, emissions would be reduced, and fuel costs would increase in comparison to a least-cost dispatch scenario.

3) The Use of Low Sulfur Fuel:

Use of low sulfur fuel such as low sulfur coal or natural gas is an option to reduce SO_2 emission. A relatively low cost alternative that will enable many utilities to meet the CAAA-90 requirements relies on burning coals that naturally have a lower sulfur content. Many studies have indicated that shifting to lower sulfur fuels is less expensive than retrofitting scrubbers on existing facilities as a means of reducing total SO_2 emissions. Such fuel switching requires some boiler modifications because lower sulfur fuels tend to produce more ash and less heat than their higher sulfur counterparts. Modifications may be needed to maintain acceptable levels of operation and performance.

4) Flue Gas Desulfurization (FGD):

In the United States, approximately 11,000 MW of utility boiler capacity has been retrofitted with post combustion flue gas desulfurization (FGD) (Emmel et al., 1988; Emmel and Maibodi, 1989; Heslin and Hobbs, 1991). FGD technology uses an alkaline slurry to absorb sulfur dioxide in the flue gas, producing sodium or calcium sulfur compounds. These technologies are categorized as wet or dry, depending on the state of the reagent as it leaves the absorber. Wet FGD systems represent the majority of FGD systems in use for new plants and retrofit applications today. The typical wet system uses a calcium-based reagent (limestone or lime) and is designed for 80% to 95% SO_2 removal efficiency. The wet FGD process yields a by-product that contains substantial moisture. Disposal is required for these by-products, and they may require

dewatering prior to disposal. The reaction products from the dry injection processes including spray drying are collected in a dry form with fly ash. In the recovery processes, sulfur is recovered from flue gas in the form of elemental sulfur, sulfuric acid, or liquid SO_2 .

5) Use of Clean coal technology:

Commercial coal cleaning facilities in the U.S. use physical cleaning techniques to reduce the mineral matter (ash) and pyritic sulfur content (Emmel et al., 1988; Emmel and Maibodi, 1989; Heslin and Hobbs, 1991) . As a result, cleaned coal has a higher energy density and less variability than feed stock coal so that power plant efficiency and reliability are improved. A side benefit to these processes is that SO_2 emissions can be somewhat reduced. The SO_2 removal rate ranges from 13% to 41%, depending on the type of coal the generation unit is using.

6) Purchasing Allowances:

This legislation creates a market in emission allowances, which allows units whose allowances exceed their SO_2 emissions to trade their excess allowances to other units which have too few allowances. This creates opportunities for profitable sales of excess allowances by utilities who can cheaply control their emissions to other utilities with high control costs.

Many existing models can be used to evaluate various emissions reduction strategies by simulating each strategy. However, there is presently no model to solve

the problem of optimizing capacity expansion and emission controls; i.e., to construct a comprehensive, cost-effective emission capacity control strategy for electric utilities, subject to demand and other operating restrictions. This study is intended to develop such a model while considering the uncertainty of the demand growth, the uncertainty of the price of allowances, and changing fuel prices. For our model, the objective cost function will be the total costs: the initial investment cost due to either of installation of emission control equipment or building a new power plant, the operating cost of the generation system, and the cost of SO₂ emissions.

Scope

This study begins with a survey of existing models that can be used to evaluate the cost-effectiveness of a compliance strategy. Past attempts to deal with the emission compliance problem are investigated.

The Emission Capacity Controls Problem (ECCP) is formulated as a NonLinear Programming (NLP) problem in Chapter 3. Chapter 4 presents solution procedures to solve the ECCP. Chapter 5 outlines the Cumulant method of calculating the expected energy generation in the generating system by probabilistic simulation. Also in that chapter we present a closed-form and computationally efficient expression for obtaining the derivatives of the expected energy generated by each plant based on the cumulant method. An implementation of the ECCP model is discussed in Chapter 6. In addition, Chapter 6 will include the results of three test problems run using data based on the characteristics of Gainesville Regional Utilities

and Electric Power Research Institute (EPRI). Finally, in Chapter 7 we will summarize the contributions of this dissertation, discuss the limitations of the proposed model, and provide suggestions for future research.

CHAPTER 2

LITERATURE REVIEW

While a large volume of research exists with regard to the Clean Air Act Amendments of 1990 (CAAA-90), very little work has been done related to the cost of implementation or to the strategies involved in complying with Title IV, the acid rain reduction provision of the CAAA-9. However, there has been substantial research performed, and models created relative to optimum operation of power plants based on factors including SO₂ emissions. The objective of this chapter is to review these models and to point out the need for a comprehensive model that overcomes the drawbacks of previous models and can systematically evaluate all available compliance options and determine an optimal compliance plan.

Emission Reduction Evaluation Models

This section reviews models and methods to evaluate various emission reduction strategies for electric generating units. These models include linear programming (LP) models, nonlinear programming (NLP) models, dynamic programming (DP) models, screening models, and emissions reduction supply curve models.

Linear, Nonlinear and Dynamic Programming Models

Linear programming applications for power system, economics, and environmental resources planning have received considerable use. Linear programming models have the ability to incorporate information about environmental systems in a single summary model and process this information to generate optimal solutions.

Many linear programming models for SO₂ abatement have been constructed. Examples include those of Masse (1962), Bessiere and Masse (1965), Fortin and McBean (1983), Ellis et al. (1983), and Alcamo et al. (1987). Many of them are large-scale linear programs, which minimize the cost of meeting power demand, subject to emission constraints, for multistate regions or the entire United States. Most of these models include decision variables for plant fuel use, plant output, and equipment retrofits. They are most useful for analyzing national energy and environmental policies and for projecting the effect of policy changes upon the coal market. Their ability to provide detailed analyses for an individual state or utility is limited. They treat power system operation in a simplistic fashion. The continuous variation of power demands over the year is modeled using two to five discrete levels of demand. Individual generation units, which vary greatly in terms of their efficiency, cost, and emissions, are often aggregated into a small number of categories. "Forced outages" of generation units (random, unplanned outages which make units available 95% to 80% of the time) are not treated as random events but are instead modeled by decreasing the effective capacity of the unit. A constraint for

the minimum generation for a unit (called must run constraint which states that the generating unit must produce at least some amount of output) is usually not considered. Another difficulty with linear programming (LP) models is the large number of constraints encountered in any realistic formulation of a problem.

These shortcomings imply that these LP models are too simplistic to yield credible estimates of impacts of policies upon particular generation plants. Further, their analysis of emissions dispatching would be weak because of the many simplified assumptions about power operation that they make.

Delson and Shahiehpour (1992), demonstrated that a linear programming technique has been applied effectively to power system engineering problems. Delson illustrated the effectiveness of LP and integer linear programming in three different areas-- generation scheduling, loss minimization through allocation of reactive power supply, and planning of capital investments in generation equipment. The conclusion of Delson's study is the recommendation that power system planning models should incorporate financial flow with the linear programming approach to capital budgeting. He demonstrated his findings with three different examples of linear programming models.

In responding to Belgium's recent national and community legislation that regulates the emission level of SO_2 , Hecq and Kestemont (1991), constructed a linear programming model to minimize the overall cost of compliance with the legislation. The strategies for reduction of SO_2 emissions are based on the application of three control technology options: 1) the "do nothing" option; 2) the use of clean coal; and

3) flue gas desulfurization (FGD). On the other hand, the strategies for controlling nitrogen oxide (NO_x) involve three control technology options: 1) the "do nothing" option; 2) the use of a low NO_x burner; and 3) the selective catalytic reduction (SCR). One major difference between the Belgian regulations and the CAAA-90 is the introduction of the allowance system. In Hecq and Kestemont's model, two major variables are not accounted for; one is the allowance system and the other is the forced outages of the power plants.

Huang and Hobbs (1994) constructed a nonlinear integer model that uses probabilistic production costing to simulate system generation. The formulation of the model is based on probabilistic production costing techniques which are now a standard approach used by utilities and regulatory commissions to forecast generating unit output and costs. The model is applied to a utility system to find the optimal compliance plan under a set of prices of emission allowances. This model has some points that it does not cover, such as those listed below.

- 1) The model considers only one period to minimize the total costs of generating electricity instead of covering a multiple period of time; i.e. Phase I from 1995 to 2000 and Phase II 2000 to 2010.
- 2) Retiring a unit and unit idling are not considered as an option to reduce emissions.
- 3) Uncertainties of the price of allowances are ignored. The price of the allowance is assumed as fixed.
- 4) Uncertainties of fuel prices are ignored.
- 5) DSM is not considered as an option to reduce emissions.

- 6) A constraint for the minimum generation for a unit is not considered.
- 7) Expansion of operation is not considered. The option of building a new power plant was not considered in Hobbs formulation.

Miller et al. (1988) used a system dynamics based model to analyze the potential of seventy-two energy conservation programs in the residential, commercial, and industrial sector. They studied the impacts of both modest and more aggressive conservation programs and analyzed a range of SO₂ reduction requirements. The model (Integrated Electric Utility Planning Model) was originally constructed as part of a study of the impact of conservation on an emission control program for the Midwest (Ford and Nail, 1985). The model is written in DYNAMO, a computer language that implements the system dynamics methodology. The model is intended for use as a strategic planning tool. The model can test a wide variety of policy options and can predict the general impact of any alternative on the entire supply-demand system. The model simulates the function of all key aspects of the supply and demand for electricity in a single integrated framework. As a consequence of structuring the model to include so many different elements of the consumer-utility system, no single sector can be modeled exhaustively in scope or detail. Although the model is supported by extensive plant and utility-specific documentation, some aggregation of inputs and results is required which makes the results less accurate.

Individual SO₂ Reduction Technologies

In this section, we briefly summarize some of studies that have been conducted to address the feasibility of various SO₂ reduction technologies in compliance planning.

In response to increased availability of natural gas and to the CAAA-90, utilities are considering building gas-fired generation and adding gas-burning capability to existing units (Tabors and Flagg, 1986; Tabors, 1989). Co-firing of coal and gas may be attractive to many utilities. Tabors (1989) modeled the economic viability of seasonal fuel-switching from coal to natural gas in existing coal-fired utilities as a means of reducing the adverse effects of SO₂. Fuel switching is compared with the costs and environmental benefits of using flue gas desulfurization (FGD) equipment. Natural gas was chosen as a possible alternative fuel for four reasons according to Tabors:

- 1) Natural gas is clean burning. It produces no SO₂.
- 2) Initial capital investment for retrofitting is low, particularly compared with the capital cost of installing FGD.
- 3) An existing plant can be relatively easily and quickly retrofitted to burn natural gas.
- 4) Supplies of natural gas are readily available at the present time and, it is generally felt, will continue to be available at least through the next five years.

Hobbs et al. (1992) proposed a decision tree-based model for quantifying the

relative economic value of the flexibility of having both gas and coal firing. Hobbs modeled the problem as stochastic dynamic programming. This approach can be used to estimate the contribution that demand side management, short lead-time power plants, and bulk power purchases could make to system flexibility. They concluded that co-firing is cheaper than switching to low sulfur coal. Co-firing enables the utility to take advantage of low gas prices. However, the exact value of the benefit of flexibility depends highly on subjective judgments concerning probabilities of fuel prices, demand changes, and other factors. Thus, Hobbs' method is best interpreted as a way of showing the economic implications of such judgments, rather than a scientific technique that yields objective results. In the model the flexibility of natural gas co-firing in comparison to using low sulfur coal to comply with requirements of the CAAA-90 was found to be cost effective. One of the drawbacks of the dynamic programming of Hobbs et al. is an exponential increase in the computational effort with system dimensions. This curse of dimensionality limits the applicability of dynamic programming algorithms to problems with a small number of variables (typically three or four).

Heuristic Approaches

Heuristic approaches such as the screening method and emissions reduction supply curves have been used to evaluate compliance plans.

The idea of the screening method is to reduce the myriad of potential

compliance options using screening criteria. These criteria may be technical, financial, or economic.

The Emission Reduction Supply Curve method was proposed in the Draft Guidebook of the Electric Power Research Institute (Temple, Baker & Sloane, 1990). The steps to implement the methods are 1) obtain incremental tons of SO₂ emissions removed and the associated incremental costs by each option; 2) calculate the incremental cost per ton of SO₂ removed; 3) rank options in order of increasing \$/ton values; 4) plot these options in the emission reduction supply curve; and 5) select the options in order of increasing \$/ton values until the total emissions reduction requirements are met. This method is illustrated by an example from Temple, Baker & Sloane (1990). The major disadvantage of this method is that it considers each option in isolation. It is based on an analysis of generating units one at a time. A fixed capacity factor is assumed, which ignores interactions with emissions dispatch.

McKee (1993) constructed a screening model of the allowance market using five utility systems. Each plant receives allowances for Phase I, but will require more allowances to meet the Phase I SO₂ emission limits of Title IV of CAAA-90. A control option is available to each plant to reduce SO₂ emissions at an estimated cost. In McKee's model, three options are available for the utility to achieve compliance. First, the utility may continue its same operation or buy allowances as needed from different sources. Second, the utility may use the control option to reduce emissions

to a level where allowances received exactly match the utility's expected emissions. Finally, the utility may over-reduce emissions by controlling options and selling allowances to other utilities. McKee used enumeration techniques to find the minimum cost for each utility to comply with the act. As above, considering each control option separately appears to be one of the major drawbacks of McKee's model.

Wille et al. (1993) discussed how the CAAA-90 will alter system dispatch and marginal cost. One effect of the 8.9 million ton cap on emissions is to increase the cost of energy from coal and oil fired power units. Wille illustrated the cost increase by providing examples of two utilities and their cost of energy before and after the act. Since January 1, 1995, when the CAAA-90 took effect, a utility must include another cost to generate electricity, that is the cost to comply with the Clean Air Act Amendment. The criterion for determining system dispatch has to change to accommodate emission costs as well as the production costs if a utility is to minimize its overall costs. Finally, the predicted price of allowances becomes as important as the prediction of fuel cost in order to determine the system dispatch.

Andrew (1993) introduced a simple system-level emissions model to design efficient integrated resource planning rules. In his model, Andrew examines extending the life of existing power plants, building new power generators, and developing demand-side management in order to identify cost effective pollution reduction strategies. He compares these three different methods to find the minimum cost of

generating electricity to meet the projected demand. The model has a simple production function that relates total annual pollution to average and marginal emission rates and the quantity of electric power produced by each method. Andrew compares different strategies, finding the cost of each and determining the best strategy according to the level of pollution. In Andrew's model, the allowances system was not considered, nor did he consider installing scrubbers as an option. Also, the model considered a fixed capacity of each power plant ignoring the forced outages.

Summary

In this chapter, we review various models to evaluate the cost effectiveness of emissions reduction options and past attempts to deal with the SO₂ emissions compliance problem. Linear/nonlinear and dynamic programming models have been developed by focusing on decisions made within one period. However, these models yield simplified approximations that make them unattractive for detailed planning. Some of these approximations include downward adjustments of generating unit capacities to account for random forced outages; disregarding of important operating considerations for generating units, such as "must run" constraints and heat rates that depend on unit output; and aggregation of units into a few categories of generating hardware.

The solutions obtained by screening and heuristic methods are not necessarily optimal. This is due to the fact that methods often consider each option in isolation. The screening method will not necessarily give the optimal solution because the combination approach might give a better solution. Another reason that the screening method is not an optimal method is that there are important interactions between strategies which are ignored if individual measures are considered in isolation. In order to obtain a realistic and practical plan for the utilities, a model has to consider the broader range of compliance and expansion options that are available for the utilities. Furthermore, the variety of emissions reduction technologies, demand side management, and the new allowance system of trading must consider all these options simultaneously in order to come up with a realistic compliance plan.

In conclusion it appears that we lack an optimization model that can systematically examine all available compliance and expansion options and minimizes the cost over a specified period of time. The main objective of this dissertation is to present such a model and provide a solution based on nonlinear programming techniques and partitioning principle. We will accomplish this while considering the variation of energy demand over the years, the uncertainty in the price of allowances, the changes in the price of fuels, and the constraints upon the operation of individual units. To do this, the problem will be formulated in Chapter 3 as a nonlinear programming problem. A solution procedure based on the partitioning principle will be proposed in Chapter 4. In Chapter 5, we will outline the Cumulant method of

calculating the expected energy generation in the generating system by probabilistic simulation. In addition, we will present a closed-form and a computationally efficient expression for obtaining the derivatives of the objective function based on the cumulant method. Numerical examples will be presented to illustrate the application of the model.

CHAPTER 3

EMISSION CONTROLS PROBLEM FORMULATION

This chapter formulates the Emission Capacity Control Problem (ECCP). The problem is defined in the section below. The optimization of power system operation over periods of time requires a probabilistic analysis-based simulation model which combines reasonable accuracy with high computation speed. The probabilistic production costing method is used to calculate the generation unit output and the cost of the output. This is a standard method used by utilities. A formulation of the multi-year emission control problem is presented in the final section.

Emission Capacity Control Problem (ECCP) for Electric Utility

The emission capacity control problem that this thesis intends to solve can be stated as follows. An electric utility generation system consisting of a number of generating units wishes to develop an optimal compliance plan in order to comply with the new acid rain law and plan for any expansion of their units. As described in Chapter 1, this utility has several options, such as fuel switching, FGD, energy conservation, and emissions trading. Each generating unit can use many types of emissions removal technologies. Different technologies have different investment costs

and have different effects on the generating unit's fixed and variable operating costs and emission rate. For instance, FGD has a 90% SO₂ emissions removal rate, while its capital cost is, on the average, about \$200/KW (Tabors, 1989). On the other hand, limestone injection multistage burners (LIMB) cost a lot less but remove only about 50% of SO₂ emissions. The problem that the utility faces is to find a retrofit option for each generation unit such that the overall cost is minimized. This cost includes the cost of investment, the operating cost, and the cost of emissions. The overall cost should be minimized under reliability constraints, meeting demand, and other operating restrictions (such as the operating level cannot exceed its capacity), while recognizing opportunities for emissions dispatch.

The solution of the emission control problem yields an emissions compliance plan that minimizes the overall system cost. To solve the problem, a mathematical formulation is developed in the final section of this chapter.

Probabilistic Production Costing

The cost that is to be minimized in the multi-year emissions control problem includes the system's operating cost, the expense of emissions allowances, and the investment cost. This section introduces the probabilistic production costing algorithm to evaluate a generating unit's output, its operating cost, and the system's expected unserved energy.

The application of optimizing techniques to power system problems requires that a suitable method be available for simulating the operation of various generating power plants over a given period of time. A large proportion of the total power system cost depends on the amount of energy generated by the different power units. Therefore, it is essential to adopt a fast and accurate method of operational simulation to estimate the generating level of each unit in order to decide which units to run.

The calculation of the distribution of energy production is complicated by the forced outages of the generating units. Thus, the problem cannot be solved by treating individual generating plants in isolation, because each plant must be operated to achieve a maximum overall economy for the system as a whole. It is necessary to examine the total generating system before the contribution of any one generating plant can be determined.

The basis of the simulation method is the use of probability distributions to describe the system loads and the generating unit forced outage performance, and the combination of these distributions to obtain the mathematical expectancy of the amount of energy generated by each unit in the system (Booth, 1972). The original formulation of this method is described by Baleriaux et al. (1967), Booth (1972), and Bloom (1983).

If we assume that all power generating units are available all the time, then the energy generated by each unit could be shown as in Figure 1. The probability distribution of the load levels experienced by the power system may be shown in a Load Duration Curve (LDC) like the one in Figure 1.

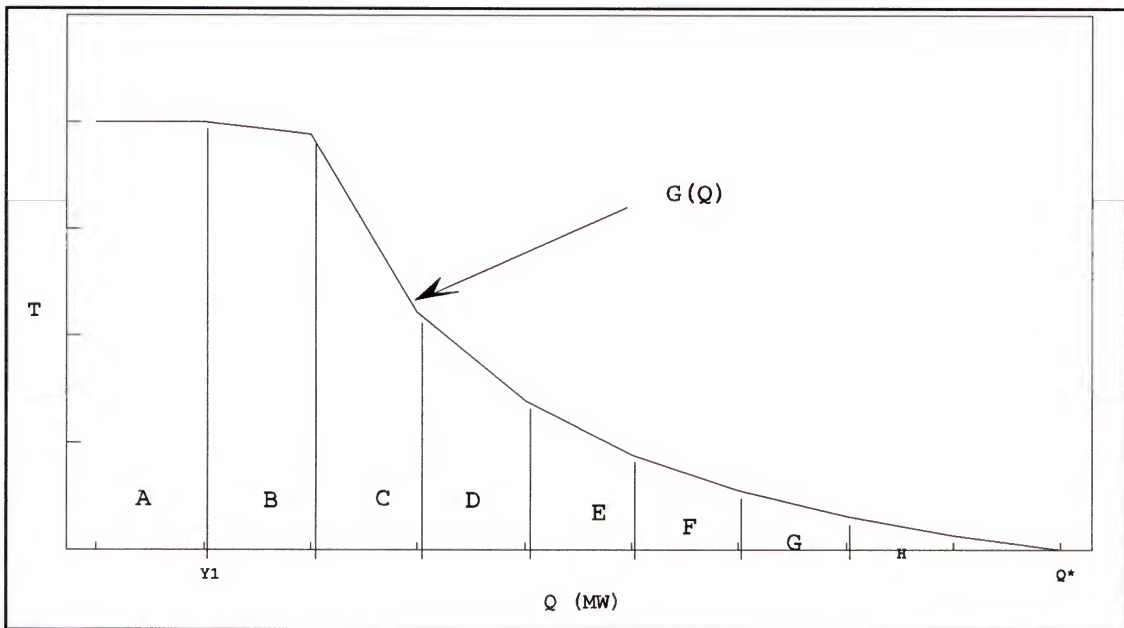


Figure 1 Units 100% available

Unit A is utilized first since it has the least expensive operating cost. Unit B would be utilized second since it has the second least expensive operating cost and we keep loading units according to their operating cost till we meet the demand. The energy generated corresponds to areas under the LDC, as shown in Figure 1. Since the plants will not be available 100% of the time, those at the lower end of the loading order are limited to less than 100% load factor, and the plants higher in the loading order must correspondingly generate more energy. The Plant Duration Curve (PLC) shown in Figure 2 relates the position in the loading order to the percentage of time that a plant in this position would be required to generate power.

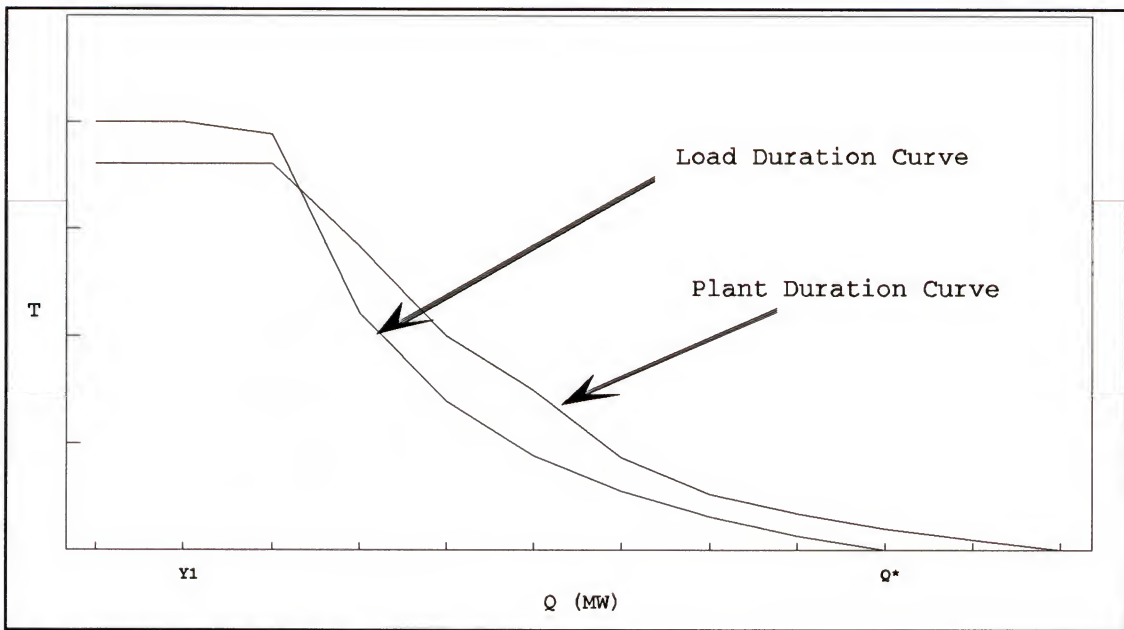


Figure 2 Units less than 100% available

Let

Y^i	Utilization level of the i th plant, MW
i	Index of plant in merit order (see next section)
I	Number of plants
Cap_i	Capacity of plant i
F_i	Operating cost of the i th plant, \$/MWh
q_i	Forced outage rate of i th plant
p_i	Availability of the i th plant
S_i	Sulfur oxide emission rate of unit i
$EG_i(Y)$	Expected generation of unit i given vector Y , MWh
$EUE(Y)$	Expected unserved energy given vector Y , MWh
U^i	Cumulative utilization of the first i plants in merit order

U^{i-1}	Loading point of the i th plant
G_i	Equivalent load duration function faced by the i th plant

The availability of unit i given the outage rate of that unit can be calculated by

$$p_i = 1 - q_i \quad (3-1)$$

The plant loading points are defined by the equations

$$\begin{aligned} Y^i &= U^i - U^{i-1} & i &= 1, 2, \dots, I \\ U^0 &= 0 \end{aligned} \quad (3-2)$$

Merit Order

The optimal way to operate a generating system consisting of thermal plants to meet a given instantaneous load of Q MW is to load each plant up to its capacity in order of increasing operating cost until the total output equals the demand. This scheme is called "Merit Order Operation".

If we have I generating units whose operating costs are $F_1, F_2, \dots, F_I$, the merit order indices i are defined in each period so that $F_1 \leq F_2 \leq \dots \leq F_I$

The Deterministic Case (No Failures Considered)

In the deterministic case, we assume that the units do not fail. Let $G(Q)$ be the continuous load duration function for each level of load Q , the expected time during which the load exceeds Q .

Assume that the function G is continuous, then the expected energy generated

by the i th plant of the merit order is given by (Bloom, 1983)

$$EG_i = \int_{U^{i-1}}^{U^i} G(Q) d(Q) \quad (3-3)$$

where U^i is the cumulative utilization of the first i plants in the merit order.

The Non-Deterministic Case (Failures Considered)

When plant failures are considered, the situation is more complicated, and the probabilistic simulation model of Booth (1972) is used. Plant operation is assumed to follow an alternating renewal process, and the plant availability p^i is defined as the expected fraction of time that the plant is available for operation. It is assumed that a plant's failures are independent of the load and the failures of other plants.

Assuming that the plant is operating, the load duration curve for the first plant in the merit order is just the system load duration curve (Figure 3), $G(Q)$. Hence the expected energy produced by the first plant in a given period of time is

$$EG_1 = p_1 \int_0^{Y_1} G(Q) dQ \quad (3-4)$$

where Y_1 is the capacity of plant 1

Conditional probability is used to derive the expression for the expected energy produced by the second plant in the merit order. The second plant faces two

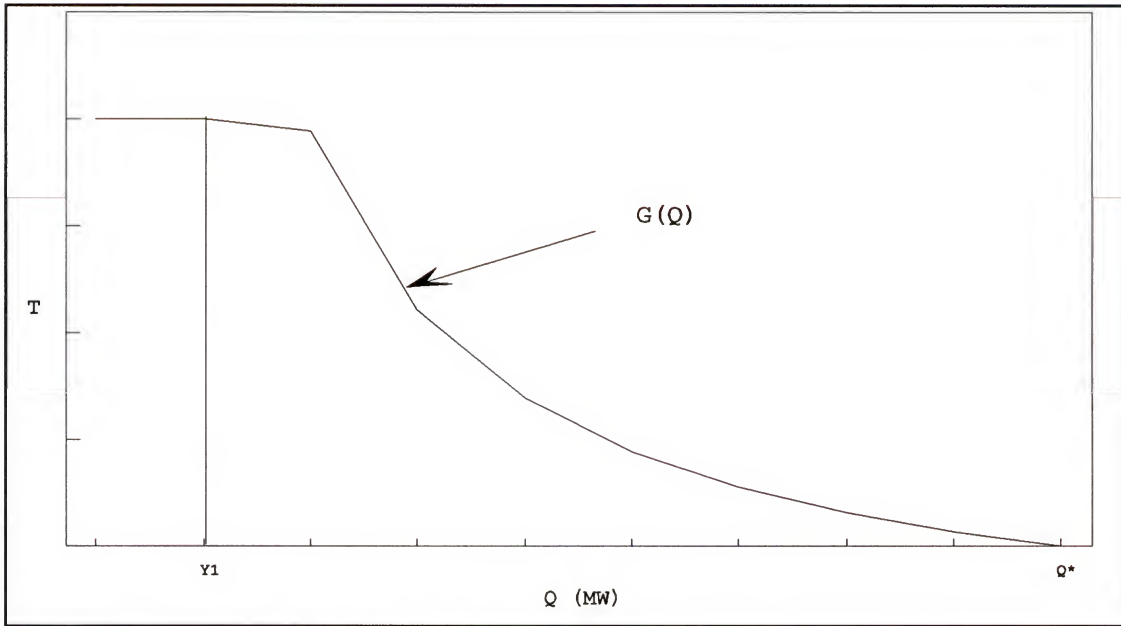


Figure 3 The first plant faces the system load duration curve.

alternative situations depending on whether or not the first plant is operating. If the first plant is operating (Figure 4), the second plant is placed on-line after the first and produces energy up to its capacity.

$$p_2 \int_{Y_1}^{Y_1+Y_2} G(Q) dQ \quad (3-5)$$

where Y_2 is the capacity of plant 2

This situation occurs with probability p_1 .

$$p_1(p_2 \int_{Y_1}^{Y_1+Y_2} G(Q) dQ) \quad (3-6)$$

where Y_2 is the capacity of plant 2

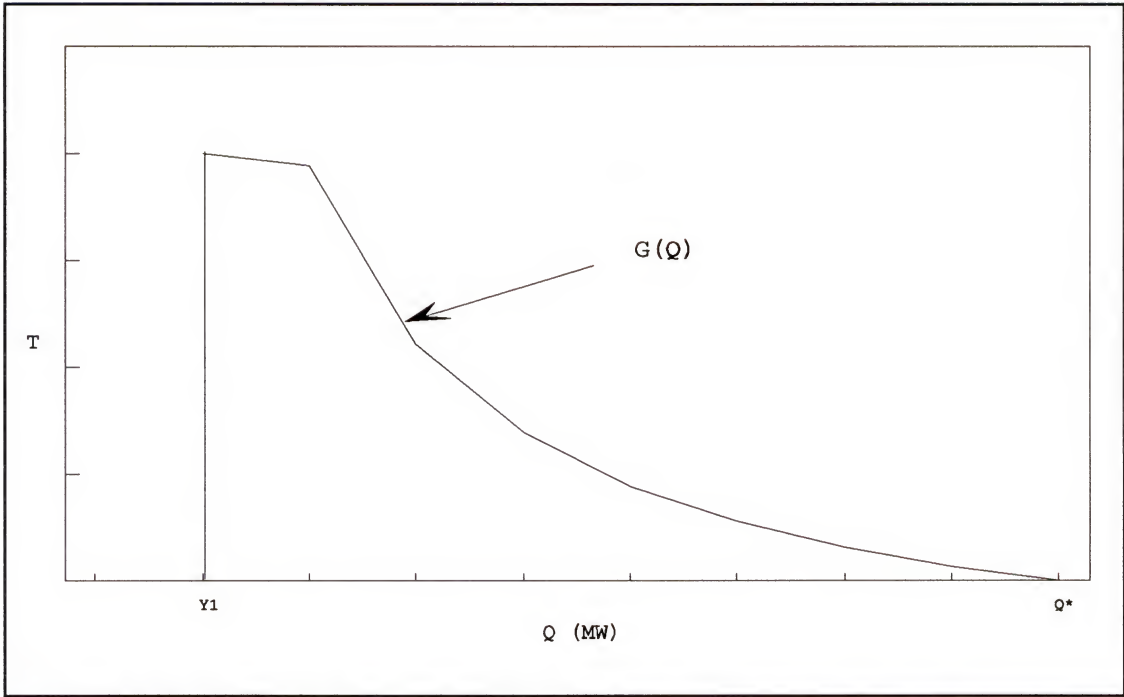


Figure 4 If the first plant is operating, the second plant faces only the load not served by the first.

If the first plant is not operating (Figure 5), the second then becomes first in the merit order and produces expected energy as formulated in the following equation.

$$p_2 \int_0^{Y_2} G(Q) dQ \quad (3-7)$$

Y_1 is the capacity of plant 2

This situation occurs with probability q_1 . Thus, the expected energy produced by the second plant is

$$EG_2 = p_2 \left(p_1 \int_{Y_1}^{Y_1+Y_2} G(Q) dQ + q_1 \int_0^{Y_2} G(Q) dQ \right) \quad (3-8)$$

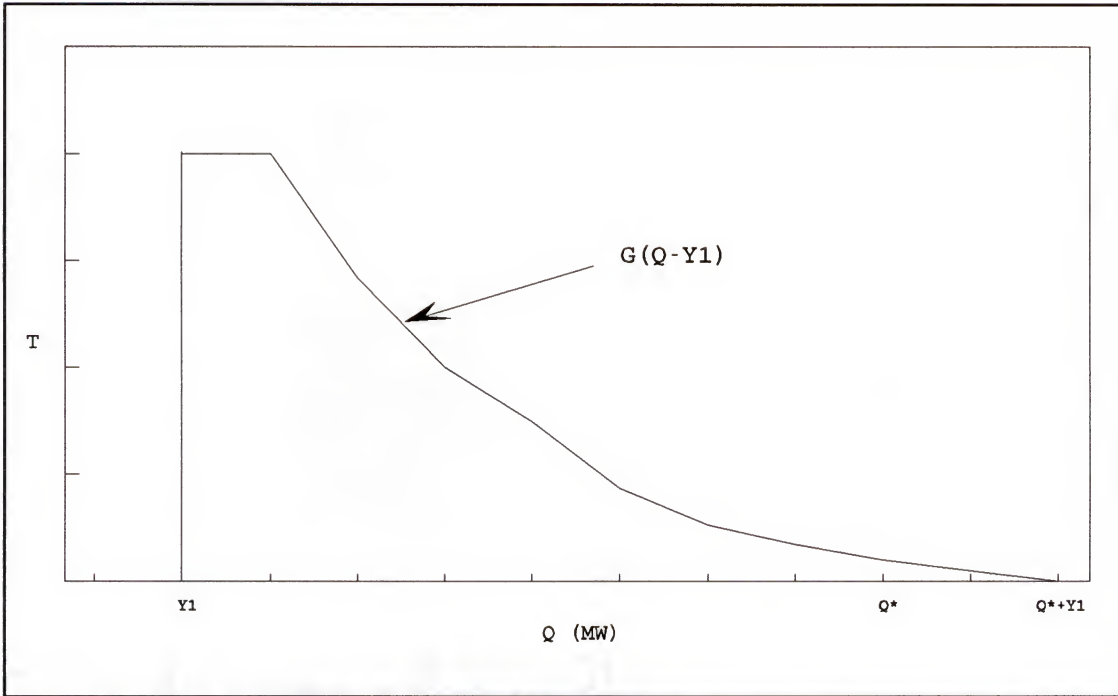


Figure 5 If the first plant fails, the second plant faces the entire system load duration curve. This event has probability q_1 .

$$EG_2 = p_2 \int_{Y_1}^{Y_1+Y_2} (p_1 G(Q) dQ) + q_1 G(Q-Y_1) dQ \quad (3-9)$$

The term in parentheses in (3-9) is the equivalent load duration curve faced by the second plant.

Using this same conditional probability argument for each successive plant (Figure 6), the equivalent load duration curve for the $(i + 1)$ st plant in the merit order is defined by

$$G_{i+1}(Q) = p_i G_i(Q) + q_i G_i(Q - Y_i) \quad (3-10)$$

where $G_1(Q) = G(Q)$

This recursive relationship is known as probabilistic simulation. The expected energy delivered by the i th plant in the merit order during the given time period is then

$$p_i \int_{U_{i-1}}^{U_i} G_i(Q) dQ \quad (3-11)$$

where
 $Y_i = U_i - U_{i-1}$
 $U_0 = 0$

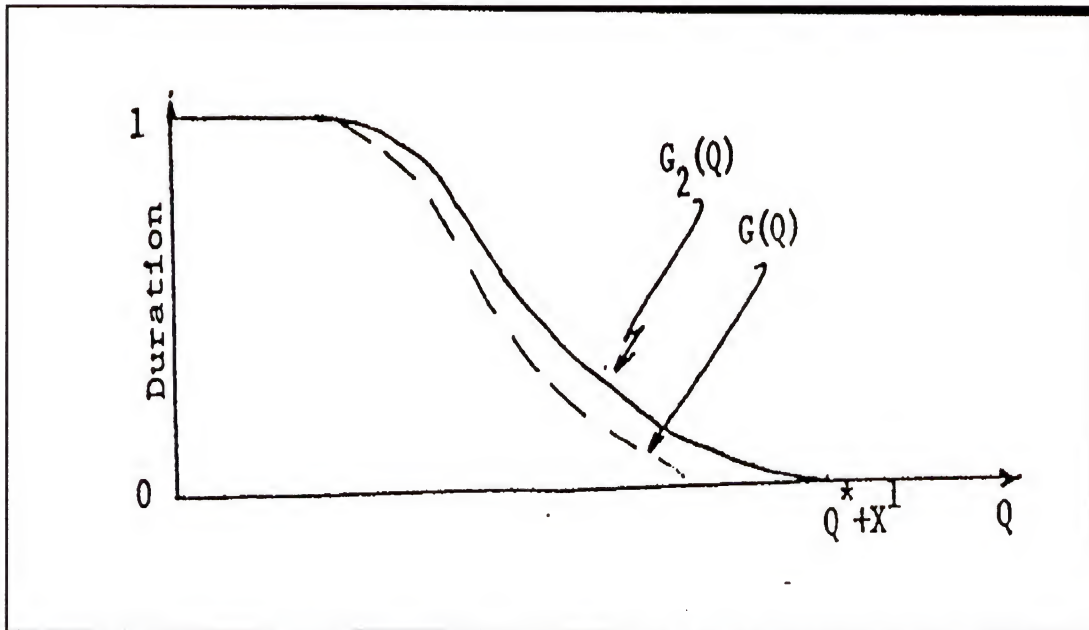


Figure 6 The equivalent load duration curve faced by the second plant is the sum of these two curves weighted by their probabilities.

Another approach to the above conclusion can be shown as follows. Define an outage state σ to be the occurrence of a random event in which a given subset of the plants I_σ are available for operation and the I_{σ^c} are down for repairs. Given I plants,

there are 2^I possible outage states. The probability of a particular state σ occurring is given by

$$\phi_{\sigma} = \prod_{i \in I_{\sigma}} p_i \cdot \prod_{i \in I_{\sigma}^c} q_i \quad (3-12)$$

Within a particular outage state, merit order operation of the available plants is optimal. Thus, define the cumulative utilization in state σ of the first i plants by

$$Y_i \Theta_{i\sigma} = U_{\sigma}^i - U_{\sigma}^{i-1} \quad i = 1, 2, \dots, I \quad (3-13)$$

$$U_{\sigma}^0 = 0$$

where the constants $\Theta_{i\sigma}$ are defined by

$$\Theta_{i\sigma} = \begin{cases} 1 & \text{if plant } i \text{ is available in state } \sigma \\ 0 & \text{if not} \end{cases} \quad (3-14)$$

As in the deterministic case, the energy produced by plant i in state σ is

$$EG_i = \int_{U_{\sigma}^{i-1}}^{U_{\sigma}^i} G(Q) d(Q) \quad (3-15)$$

which is zero if the plant is not available because $U_{\sigma}^i = U_{\sigma}^{i-1}$. If plant i is available in state σ , this expression can be written as

$$EG_i = \int_{U^{i-1}}^{U^i} G(Q - U^{i-1} + U_\sigma^{i-1}) d(Q) \quad (3-16)$$

The term $U^i - U^{i-1}_\sigma$ is the total utilization level of the plants on outage below i in the merit order. Then the expected energy produced by plant i is

$$EG_i = \sum_{\sigma=1}^{2^I} \phi_\sigma \int_{U_\sigma^{i-1}}^{U_\sigma^i} G(Q) d(G) = \sum_{\sigma \in \epsilon} \phi_\sigma \int_{U^{i-1}}^{U^i} G(Q - U^{i-1} + U_\sigma^{i-1}) d(Q) \quad (3-17)$$

where the latter summation is taken over all states σ in which plant i is available.

Define the equivalent load duration function G , faced by plant i as the expectation over all outage states

$$G_i(Q) = \sum_{\sigma=1}^{2^I} \phi_\sigma G(Q - U^{i-1} + U_\sigma^{i-1}) \quad (3-18)$$

Then the expected energy produced by plant i can be written as

$$EG_i(y) = \sum_{\sigma=1}^{2^I} \phi_\sigma \int_{U_\sigma^{i-1}}^{U_\sigma^i} G(Q) d(Q) = p_i \int_{U^{i-1}}^{U^i} G_i(Q) d(Q) \quad (3-19)$$

Multiplying the expected energy of each plant by its operating cost and summing over all plants gives the expected operating cost expression

$$EF(Y) = \sum_{i=1}^I F_i p_i \int_{U^{i-1}}^{U^i} G_i(Q) d(Q) \quad (3-20)$$

Because the number of outage states grows exponentially with the number of plants, it is not convenient to deal with the states explicitly, as in the above equation. Instead, the equivalent load duration functions can be computed recursively by using the convolution equation

$$G_{i+1}(Q) = p_i(G_i(Q) + q_i G_i(Q - Y^i)) \quad i = 1, 2, \dots, I \quad (3-21)$$

where $G_i(Q)$ is the original system load duration curve $G(Q)$.

This equation is the heart of the probabilistic simulation model, derived by Baleriaux et al. (1967), and Booth (1972). Defining the equivalent load duration functions recursively allows the outage states to be dealt with implicitly.

In outage state σ , the amount of unserved energy is given by

$$\int_{U_\sigma^I}^{\infty} G(Q) d(Q) \quad (3-22)$$

where U_σ^I is the total utilization of all the available plants. The expected unserved

energy over all outage states can be written as

$$EUE(y) = \int_{U^I}^{\infty} G_{I+1}(Q) d(Q) \quad (3-23)$$

Result 1

$$EG_i(y) = p_i \int_{U^{i-1}}^{U^i} G_i(Q) d(Q) \quad (3-24)$$

Result 2

$$EUE(y) = \int_{U^I}^{\infty} G_{I+1}(Q) d(Q) \quad (3-25)$$

Result 3

$$EUE(y) = T.E(D) - \sum_{I-1}^i EG_i \quad (3-26)$$

Result 4

$$\begin{aligned} \text{Total operating cost} &= \sum_i F_i EG_i \\ \text{Total Emission} &= \sum_i S_i EG_i \end{aligned} \quad (3-27)$$

The Formulation of Capacity Controls Problem

The objective of this section is to present a formulation of the multi-year emission control problem.

Consider a utility with many generation units $1, 2, 3, \dots, I$. The utility can fit each unit i with a retrofit option j . The retrofit options include fuel switching, flue gas desulfurization, clean coal, do-nothing, etc. There will be J_i available retrofit options for each unit i . Assume the price of allowances for each period is given as P_t (\$/ton).

The emission control problem can be stated as follows: Given

- 1) a utility has I power units.
- 2) the type, capacity and cost of building a new power plant are known.
- 3) the price of allowances for each period is known.
- 4) the retrofit options and their effect on each unit is known.
- 5) the operating cost for each unit is known.
- 6) the operating cost for each unit after the retrofit is known.
- 7) the cost of each retrofit on each unit is known.
- 8) the penalty cost for the unserved energy is known.
- 9) the cost of over-complying with the CAAA-90 is known.
- 10) the reliability level for all units is known.
- 11) the capacity of each unit before and after retrofit is known.
- 12) the number of allowances given to a utility is known.
- 13) the minimum run level of each unit is known.

Our objective is to find out what is the most cost effective way to both comply with the CAAA-90 and the capacity expansion to minimize the cost of generating electricity which includes the cost of investment, the cost of operation, and the cost of emissions.

We define unit ij to be the unit i with retrofit option j installed. The problem can be stated as: given all units ij where $1 \leq i \leq I$, and $1 \leq j \leq J^i$, find I units such that the cost of investment, the operating cost and the emission cost are minimized subject to the reliability constraints, capacity constraints, minimum run constraints and the constraints of meeting the demand.

Define:

Y_{ijt}	Decision variable ,utilization level of the i th plant with option j in period t , MW
i	Index of plant in merit order
I	Number of plants
Cap_{ijt}	Capacity of plant i with option j in period t
F_{ijt}	Operating cost of the i th plant retrofited with j option in period t , \$/MWh
q_{it}	Forced outage rate of i th plant in period t
p_{it}	Availability of the i th plant in period t
S_{ijt}	Sulfur oxide emission rate of unit i with option j in period t
$EG_{ijt}(Y_t)$	Expected generation of unit i with option j in period t given Y_t , MWh

$EUE_t(Y_t)$	Expected unserved energy given Y_t , MWh
X_{ijt}	Decision variable = 1 if unit ij is chosen 0 else
C_{ijt}	Investment cost when unit i is retrofited with option j
K_t	Penalty cost for unserved energy in period t
P_t	Price of allowance in period t
R_t	Desired reliability level in period t
T	Number of periods in planning horizon
NP_{ijt}	New power plant i of type j built in period t
CNP_{ijt}	Cost of building new power plant i of type j in period t
m_{ij}	Minimum percentage of unit i of option j must run in period t .

$$\begin{aligned} \text{MINIMIZE}_{x,y} \sum_{t=1}^T [& \sum_i CNP_{ijt} NP_{ijt} + \sum_{ij} C_{ijt} X_{ijt} + \sum_{ij} F_{ijt} EG_{ijt}(y_t) \\ & + \sum_{ij} P_t S_{ijt} EG_{ijt}(y_t)] \end{aligned} \quad (3-28)$$

$$\begin{aligned} Y_{ijt} &\leq Cap_{ijt} X_{ijt} \quad (1) \\ EUE_t(Y_{ijt}) &\leq R_t \quad (3) \\ Y_{ijt} &\geq m_{ijt} Cap_{ijt} \quad (4) \\ X_{ijt} &= 0, 1 \quad (5) \end{aligned} \quad (3-29)$$

To illustrate the above formulation, we will consider the following example:

A utility has three power plants number 1, 2, 3 respectively. The plant data are given

in table 3.1.

Table 3.1 Plant Data

Plant Number	Plant type	Standard Size (MW)	Forced Outage (%)	Operating Cost (\$/MWh)	Emission Rate (lb/MWh)
1	Coal	1000	20	15	60
2	Coal	400	18	20	50
3	Gas	200	8	50	0

For initial year of the planning horizon
Energy Demand = 4,800,000 MWh

Unserved Energy Constraint $\leq 3\%$ of energy demand (for all years)

Load Growth = 3%

Retrofit Data

Table 3.2 Retrofit Data

Plant Number	Retrofit type	New Size (MW)	Operating Cost (\$/MWh)	Emission Rate (lb/MWh)	Fixed Cost
1	FGD	960	20	10	2,000K
1	LSC	980	18	35	100K
2	FGD	360	28	40	1,000M
2	LIMB	350	25	25	400K
3	---	---	----	---	---

Expansion Data

Table 3.3 Expansion Data

Plant type	Size (MW)	Forced Outage (%)	Operating Cost (\$/MWh)	Emission Rate (lb/MWh)	Cost
Nuclear	500	15	10	0	15M
Gas	400	20	35	0	10M

Problem Formulation

Let

X_{ijt} = Option Considered at period t

X_{111} = Install FGD on plant 1 at period 1

X_{211} = Install LSC on plant 1 at period 1

X_{121} = Install FGD on plant 2 at period 1

X_{221} = Install LIMB on plant 2 at period 1

X_{41j} = Build plant 4 of a nuclear type at period j . $j=1,2,3$

X_{42j} = Build plant 4 of a gas type at period j . $j=1,2,3$

The price of allowance is \$300/ton

The period over which the units are dispatched is 700 hours

No minimum run constraint for this particular example

Minimize

Operating Cost (No retrofit)

$700 (15 \text{ EG}_{101} + 20 \text{ EG}_{201} + 50 \text{ EG}_{301})$

$700 \cdot 300 / 2000 (60 \text{ EG}_{101} + 50 \text{ EG}_{201} + 0 \text{ EG}_{301})$

$700 (15 \text{ EG}_{102} + 20 \text{ EG}_{202} + 50 \text{ EG}_{302})$

$700 \cdot 300 / 2000 (60 \text{ EG}_{102} + 50 \text{ EG}_{202} + 0 \text{ EG}_{302})$

$700 (15 \text{ EG}_{103} + 20 \text{ EG}_{203} + 50 \text{ EG}_{303})$

$700 \cdot 300 / 2000 (60 \text{ EG}_{103} + 50 \text{ EG}_{203} + 0 \text{ EG}_{303})$

operating Cost (install option 1)

700 (20 EG111 + 28 EG211)
 700*300/2000 (10 EG111 + 40 EG211)
 700 (20 EG112 + 28 EG212)
 700*300/2000 (10 EG112 + 40 EG212)
 700 (20 EG113 + 28 EG213)
 700*300/2000 (10 EG113 + 40 EG213)

Operating Cost (install option 2)
 700 (18 EG121 + 25 EG221)
 700*300/2000 (35 EG121 + 25 EG221)
 700 (18 EG122 + 25 EG222)
 700*300/2000 (35 EG122 + 25 EG222)
 700 (18 EG123 + 25 EG223)
 700*300/2000 (35 EG123 + 25 EG223)

EXPANSION OPTION

700 (10 EG411 + 35 EG421)
 700*300/2000 (0 EG411 + 0 EG421)
 700 (10 EG412 + 35 EG422)
 700*300/2000 (0 EG412 + 0 EG422)
 700 (10 EG413 + 35 EG423)
 700*300/2000 (0 EG413 + 0 EG423)

Retrofit and expansion costs

2000k X111+ 2000k X112+ 2000k X113
 100k X211+ 100k X212+ 100k X213
 1000k X121+ 1000k X122+ 1000k X123
 400k X221+ 400k X222+ 400k X223
 15M X411 +15 X412 +15 X413
 10M X421 +10 X422 +10M X423

Subject To

Unserved Energy in Year t Constraints

$EUE1 \leq 129000$
 $EUE2 \leq 132870$
 $EUE3 \leq 136856$

Capacity Constraints

Plant 1

$$Y_{101} \leq 700000 X_{101}$$

$$Y_{102} \leq 700000 X_{102}$$

$$Y_{111} \leq 672000 X_{111}$$

$$Y_{112} \leq 672000 X_{112}$$

$$Y_{121} \leq 686000 X_{121}$$

$$Y_{122} \leq 686000 X_{122}$$

Plant 2

$$Y_{201} \leq 280000 X_{201}$$

$$Y_{202} \leq 280000 X_{202}$$

$$Y_{211} \leq 252000 X_{211}$$

$$Y_{212} \leq 252000 X_{212}$$

$$Y_{221} \leq 245000 X_{221}$$

$$Y_{222} \leq 245000 X_{222}$$

Plant 3

$$Y_{301} \leq 140000 X_{301}$$

$$Y_{302} \leq 140000 X_{301}$$

Options Constraints

$$X_{101} + X_{111} + X_{121} + X_{102} + X_{112} + X_{122} = 1$$

$$X_{201} + X_{211} + X_{221} + X_{202} + X_{212} + X_{222} = 1$$

$$X_{411} \geq 0$$

$$X_{421} \geq 0$$

$$X_{412} \geq 0$$

$$X_{422} \geq 0$$

$$\text{ALL } Y \geq 0$$

CHAPTER 4

SOLUTION PROCEDURE

The purpose of this chapter is to find solution procedures to solve the Emission Capacity Controls Problem (ECCP). In the previous chapter, we formulated the problem as a nonlinear programming problem. Now this chapter will show a feature peculiar to the problem. That is, when all the X (the zero-one decision variables) values are given in advance, the remaining problem is the operating problem for the generation system. We will show how this allows us to apply the partitioning approach (Benders, 1962; Lasdon, 1970) to find the solution for the ECCP problem.

The underlying idea of the partition approach used in this model is to separate the ECCP into two parts: a master problem that generates trial solutions for the optimal emission capacity plan; and a subproblem that determines the optimal operating scheme for each trial-generating system. We will show that one of the advantages of the partition method is that these primal subproblems can be solved easily relative to the master problem.

Also, we will derive the dual operating problem using the Karush-Kuhn-Tucker (KKT) optimality conditions. The KKT optimality conditions for the subproblem are used to derive the Lagrangian multipliers, or shadow prices. The

economic interpretation of the dual multipliers will give us an insight into the master problem. The multiplier U , the shadow price, on the unserved energy constraint is the marginal cost of increasing R , the cost of operating the marginal plant. The operating cost of the marginal plant is the variable cost plus the SO_2 emission cost of producing electricity. The multiplier W_i on the capacity constraint is the marginal benefit/cost of increasing/decreasing the capacity of plant i . For a plant operating at a capacity, the benefit of increasing the capacity is just the difference between the cost of operating this plant and the marginal plant. This is due to the fact that adding extra capacity to a plant that operates at certain capacity will reduce the output needed from the marginal plant. For a plant operating at a capacity, the disadvantage of decreasing the capacity is just the difference between the cost of operating this plant and the marginal plant. This is due to the fact that reducing capacity to a plant that operates at certain capacity will increase the output needed from the marginal plant. On the other hand, if a plant is not utilized to its full capacity, there is no benefit to increasing its capacity.

We will show how the partition approach can be used to develop a solution procedure for the master problem. The multipliers are used to generate a master problem, which is another mathematical program.

The procedure developed from the partition principle is an iterative one. For each trial, the subproblems are solved to determine their optimal operating costs and the Lagrangian multipliers on the unserved energy constraint and on the plant capacity constraints. These Lagrangian multipliers are important in deriving a new constraint

in the master problem, which can then be solved again for a new trial plan. The procedure is repeated, alternating between the master problem and subproblems, until a solution to the ECCP is found.

In this chapter, developing a solution procedure for the ECCP formulation follows three steps. Section 4.2 discusses the formulation, properties and solution of the primal operating subproblem. In Section 4.3, the dual subproblem is derived. The strategy of this section is to propose solutions for the subproblem and its dual and to show that they satisfy the global optimality conditions. This argument not only demonstrates that the proposed solutions are optimal, but also establishes the Strong Duality Property required to justify replacing the primal subproblem by its dual in deriving the master problem. Section 4.4 describes the use of the partition principle (Benders, 1962) to solve the ECCP problem. The ECCP problem will be reformulated as a mathematical program with the operating problem incorporated as a subproblem. The dual multipliers are used to derive a master problem which is, as we will show, a linear programming problem.

Deriving the Subproblem

The problem of minimizing the cost of supplying a given demand for electricity (ECCP) that was discussed in Chapter 3 falls into two components. One of them is finding a minimum cost operating scheme for a given generating system. This can be cast as a subproblem. Therefore, the problem of optimizing the Emission Capacity Control Problem is decomposed into a subproblem and a master problem.

The operating cost (the cost of the subproblem) includes the cost of SO₂ emissions and the variable operating costs. The objective of the subproblem is to find an optimal dispatch order so as to minimize the operating cost of each unit.

Recall that the ECCP was stated in Chapter 3 as follows:

$$\begin{aligned} \text{minimize}_{x,y} \sum_{t=1}^T [& \sum_i \text{CNP}_{it} \text{NP}_{it} + \sum_{ij} C_{ijt} X_{ijt} + \sum_{ij} F_{ijt} \text{EG}_{ijt}(y_t) \\ & + \sum_{ij} P_t S_{ijt} \text{EG}_{ijt}(y_t)] \end{aligned}$$

$$Y_{ijt} \leq \text{Cap}_{ijt} X_{ijt} \quad (1)$$

$$\text{EUE}_t(Y_{ijt}) \leq R_t \quad (2)$$

$$Y_{ijt} \geq m_{ijt} \text{Cap}_{ijt} \quad (3)$$

$$X_{ijt} = 0, 1 \quad (4)$$

where

NP_{it} is a decision variable for building a new power plant i of type j at time t .

CNP_{it} is the cost of building a new power plant i of type j at time t .

X_{ijt} decision variable = 1 if unit i is retrofitted with option j at time t is selected
0 else

C_{ijt} the investment cost when unit i is retrofitted with option j at time t .

Instead of having a separate variable for building a new power plant, we will consider building a new power plant as an option available to the utility so as to minimize the total cost and meet the demand of electricity. In other words, X_{ijt} will also represent building power plant i of type j (coal, gas, nuclear) at time t . Certain

assumptions have to be imposed on the new formulation. One of these assumptions is that the new power plant is constructed with appropriate emission controls and is not considered an immediate candidate for retrofit.

So the new ECCP formulation will be as follows:

$$\begin{aligned} \text{MINIMIZE}_{x,y} \sum_{t=1}^T [& \sum_{ij} C_{ij,t} X_{ij,t} + \sum_{ij} F_{ij,t} EG_{ij,t}(y_t) \\ & + \sum_{ij} P_t S_{ij,t} EG_{ij,t}(y_t)] \end{aligned}$$

subject to

$$Y_{ij,t} \leq Cap_{ij,t} X_{ij,t} \quad (1)$$

$$EUE_t(Y_{ij,t}) \leq R_t \quad (2)$$

$$Y_{ij,t} \geq m_{ij,t} Cap_{ij,t} \quad (3)$$

$$X_{ij,t} = 0, 1 \quad (4)$$

where

$$EG_{ij}(y) = p_{ij} \int_{U^{ij-1}}^{U^{ij}} G_{ij}(\varrho) d(\varrho)$$

$$G_{(i+1)j}(Q) = p_{ij}(G_{ij}(Q) + q_{ij}G_{ij}(Q - Y^{ij})) \quad i=1,2,\dots,I$$

$$j=1,2,\dots,I$$

where $G_{ij}(Q)$ is the original system load duration curve $G(Q)$. Also, $G_{11}(Q) = G(Q)$,

P_{ij} is the availability of plant i with option j , and its forced outage rate is $q_{ij} = 1 - p_{ij}$.

The plant loading points are defined by the equations

$$\begin{aligned} Y^{ij} &= U^{ij} - U^{ij-1} & i &= 1, 2, \dots, I \\ & & j &= 1, 2, \dots, J \end{aligned}$$

$$U^{00} = 0$$

Y^{ij} is the utilization level for the i th plant with option j .

The expected unserved energy over all outage states can be written as

$$EUE(Y) = \int_{U^I}^{\infty} G_{I+1}(Q) d(Q)$$

We rewrite ECCP as

$$\text{minimize } \sum \{CX + \text{minimize } \sum \sum FEG(Y) + \sum \sum PSEG(Y)$$

subject to

$$EUE(Y) \leq R$$

$$Y \leq \text{Cap } X$$

$$Y \geq m \text{ Cap}$$

$$X = 0,1\}$$

For fixed or known values of X , for each period in the planning horizon, the inner optimization is just the operating subproblem:

Primal Subproblem

$$\text{minimize } \sum \text{FEG}(Y) + \sum \text{PSEG}(Y)$$

subject to

$$\text{EUE}(Y) \leq R$$

$$Y \leq \text{Cap } X$$

$$Y \geq m \text{ Cap}$$

The index t has been suppressed for clarity.

In this section we will develop an algorithm to solve the operating subproblem.

The operating subproblem can be written as follows:

$$\begin{aligned} \min_{S, T} & \sum \sum F_{ijt} P_{ijt} \int_{U^{ij-1,t}}^{U^{ijt}} G(Q)_{ijt} dQ + P_t S_{ijt} \int_{U^{ij-1,t}}^{U^{ijt}} G(Q)_{ijt} dQ \\ & \text{EUE}(Y_t) = \int_{U^I}^{\infty} G_{I+1}(Q) dQ \leq R_t \\ & Y_{ijt} \leq \text{Cap}_{ijt} X_{ijt} \\ & Y_{ijt} \geq m_{ijt} \text{Cap}_{ijt} \\ & Y_{ijt} = U^{it} - U^{ij-1,t} \\ & U^0 = 0 \end{aligned}$$

The optimal solution of the operating subproblem is to set $Y = \text{Cap } X$

successively for each plant in merit order until the unserved energy constraint ($EUE(Y)$) is satisfied. The last power plant to be utilized, the most expensive plant to operate, will not operate at full capacity. This plant is called the marginal plant. Only the plants that are ranked ahead of the marginal plant will operate at full load, $Y = \text{Cap } X$ if $i < n$ where n is the marginal plant. The plants that are ranked after the marginal plant will not operate, when $i > n$ then $Y = 0$.

Solving the primal subproblem is simple relative to solving the master problem, as shown. Because of the relative simplicity of this solution, we do not need an explicit and complicated algorithm to solve the nonlinear subproblem. However, it is useful to regard this primal subproblem as an optimization problem in order to construct the dual problem, and more importantly to find the shadow prices of both the unserved energy constraint (U) and the plant capacity constraints (W_i).

Algorithm to Solve the Operating Subproblem

Step 1 Sort all units, in period t , according to their operating costs. The operating costs are the variable cost, F_{ij} , and the emission cost, $P_t S_{ij}$.

$$F_1 + P_1 S_1 \leq F_2 + P_1 S_2 \leq F_3 + P_1 S_3 \leq \dots \leq F_I + P_1 S_I$$

Initialization $TN = T.D(Y) + R$

Step 2 for $i=1,2,\dots,I$

$$TN_i = TN$$

$$Cap_i = Cap_i$$

If $Cap_i \leq TN_i$ set

$$Y_i = Cap_i$$

else

$$Y_i = TN_i - \sum Y_i$$

Step 3

If the unserved energy is less than or equal desired reliability STOP.

Or this can be written as:

$$\sum Y_{ij} \geq TN$$

STOP. The optimal solution is found. Else

$$TN = R - \sum Y_i$$

Go to step 2.

The Dual Operating Subproblem

In the previous section, we derived the primal operating subproblem. Also, we developed an algorithm to solve the primal subproblem. From the optimal solution of the primal subproblem, we identify the marginal power plant. This section presents the dual operating subproblem of the primal operating subproblem and proves that both have the same optimal solution.

Recall the primal subproblem from last section

Primal Subproblem

$$\text{minimize } \sum FEG(Y) + \sum PSEG(Y)$$

subject to

$$EUE(Y) \leq R$$

$$Y \leq \text{Cap } X$$

$$Y \geq m \text{ Cap}$$

The index t has been suppressed for clarity.

Associated with the foregoing primal subproblem is the following Lagrangian dual

(Bazaraa and Shetty, 1979) subproblem

Dual Subproblem

$$\max \Theta(Y, U, W)$$

Subject to

$$U \geq 0$$

$$W \geq 0$$

$$Y \geq 0$$

where

$$\Theta(Y, U, W) = \min U[EUE(Y) - R] + \sum W[Y - CapX] + F EG(Y) + PS EG(Y)$$

In the expression for $\Theta(Y, U, W)$, the unserved energy constraint $EUE(Y) \leq R$ and the capacity constraint $Y \geq CapX$ have been incorporated in the objective function using the Lagrangian multipliers U, W . Also, note that the multipliers U, W must be nonnegative because the constraints in the primal are inequality constraints (Duality Theorem).

The dual subproblem can be written as

$$\max \min U[EUE(Y) - R] + \sum W[Y - CapX] + F EG(Y) + PS EG(Y)$$

Subject to

$$U \geq 0$$

$$W \geq 0$$

$$Y \geq 0$$

Let

$$\gamma(Y, U, W) = U[EUE(Y) - R] + W[Y - CapX] + FEG(Y) + PSEG(Y)$$

which is the objective function of the Lagrangian function of the dual subproblem. In order to find a solution to the dual problem, the Karush-Kuhn-Tucker (KKT)

conditions can be applied to the primal solution [Bazaraa and Shetty, 1979].

The optimality conditions for the dual problem are stated as follow [based on the duality theorem in Bazaraa and Shetty, 1979]:

1) The set of solution (Y, U, W) should be feasible. That is

$$EUE(Y) \leq R$$

$$Y \leq \text{Cap } X$$

$$Y \geq m \text{ Cap}$$

$$U, W \geq 0$$

The feasibility condition is satisfied since for $i \leq n$, Y^* will be always positive (merit order) and $Y^*=0$ for $i > n$.

2) Complementary Slackness

$$U[EUE(Y)-R] = 0$$

The part inside the bracket $(EUE(Y)-R)$ is always zero, since this constraint is always binding in order to minimize the total cost of the primal subproblem as shown in the previous section.

$$W[Y-\text{Cap}X] = 0$$

For $i < n$, according to the merit order algorithm the level of utilization of the plant should equal its capacity, $Y=\text{Cap}X$. We conclude that W must be greater or equal than zero. Now for $i=n$, the marginal plant will not operate at full capacity, $Y <$

CapX, so W must be zero. For $i > n$, these plants will not operate, $Y=0$, $W=0$.

The dual multipliers satisfying these conditions when $Y=Y^*$ (using KKT conditions [Strong Duality Theorem]) are

$$\partial\gamma(Y,U,W)/\partial Y_{ijt} = U \partial EUE(Y)/\partial Y_{ijt} + W_{ijt} + F \partial EG(Y)/\partial Y_{ijt} + PS \partial EG(Y)/\partial Y_{ijt} = 0$$

$$\begin{aligned} W_{ijt} &= -U \partial EUE(Y)/\partial Y_{ijt} - F \partial EG(Y)/\partial Y_{ijt} - PS \partial EG(Y)/\partial Y_{ijt} \\ W_{ijt} &= -[F_{ij} + PS_{ij}] \partial EG(Y)/\partial Y_{ijt} - U \partial EUE(Y)/\partial Y_{ijt} \end{aligned}$$

The multiplier U is associated with the unserved energy constraint. The interpretation of the shadow price of U is as follows: a small decrease in reserved margin (R) would have a marginal cost of U.

If we desire to reduce the reserved margin of unserved energy, this would require additional generation from the marginal unit, and maybe other units, the last unit that had been utilized $i=n$. The marginal cost will be the cost of operating that unit. We conclude that U would be the same as the operating cost of the last plant utilized, $F_n + PS_n = U$.

We can prove that $U=F_n+PS_n$ by using the complementary slackness and KKT conditions for the marginal plant n. The complementary slackness conditions are

$$U[EUE(Y)-R] = 0$$

$$W[Y-CapX] = 0$$

also

$$\partial\gamma(Y,U,W)/\partial Y=0 \text{ at optimality}$$

$$W_{ijt} = -[F_{ij} + PS_{ij}] \partial EG(Y)/\partial Y_{ijt} - U \partial EUE(Y)/\partial Y_{ijt}$$

At optimality, the marginal plant, n , does not operate at full capacity. This means that $Y < CapX$

for the n th plant. From Complementary Slackness (see section above) we conclude that $W_n = 0$ at $i=n$

$$W_n = -U \partial EUE(y)/\partial Y - F_n \partial EG(Y)/\partial Y - PS_n \partial EG(Y)/\partial Y = 0$$

$$U \partial EUE(y)/\partial Y = F_n \partial EG(Y)/\partial Y - PS_n \partial EG(Y)/\partial Y$$

$$U \partial EUE(y)/\partial Y = -(F_n + PS_n) \partial EG(Y)/\partial Y$$

$$\partial EUE(y)/\partial Y = - \partial EG(Y)/\partial Y \text{ at } i=n$$

so

$$U = F_n + PS_n$$

3) Y minimizes $\gamma(Y, U, W)$ over Y

The multipliers U, W are said to be optimal multipliers for the primal subproblem if (Y, U, W) satisfies the optimality conditions for some Y (Duality Theorem).

It is easy to verify that the (Y, U, W) satisfies the optimality conditions only if y is optimal in the primal subproblem. Thus the existence of an optimal multiplier presupposes y is optimal in the primal subproblem. The converse, of course, is not true. It also can be verified that if U, W are optimal multipliers, then (Y, U, W) satisfies the conditions for every optimal solution of the primal subproblem. Thus an

optimal multiplier is truly associated with the primal - more precisely, with the optimal solution set of the primal - rather than any particular optimal solution.

To show the minimality condition of 3, the KKT stationary condition ($\partial \gamma(Y, U, W) / \partial Y \geq 0$ for $i = 1, 2, \dots, I$, $\partial \gamma(Y, U, W) / \partial Y = 0$ for $Y^i > 0$) is a necessary condition for local optimal, but it is not sufficient. In order for the KKT conditions to be necessary and sufficient, the non-zero part of the dual objective function $\Theta(Y, U, W)$ has to be convex.

To show the objective function $\Theta(Y, U, W)$ is convex let

$$f(Y) = U \text{EUE}(Y_i) + F_i \text{EG}(Y_i) + PS_i \text{EG}(Y_i)$$

Recall from Chapter 3

$f(Y)$ = Both operating cost and the unserved energy cost

$$f(Y) = \sum_{\sigma=1}^{2^I} \Phi_{\sigma} [F_i \text{EG}_i(Y) + PS_i \text{EG}_i(Y) + U \text{EUE}(Y)]$$

$$f(Y) = \sum_{i=1}^n \Phi_{\sigma} F_i \int_{U_{\sigma}^{i-1}}^{U_{\sigma}^i} G(Q) d(Q) + PS_i \int_{U_{\sigma}^{i-1}}^{U_{\sigma}^i} G(Q) dQ + U \int_{U_{\sigma}^n}^{\infty} G(Q) d(Q)$$

Let

$$I_{\sigma} = \sum_{i=1}^n F_i \int_{U_{\sigma}^{i-1}}^{U_{\sigma}^i} G(Q) d(Q) + PS_i \int_{U_{\sigma}^{i-1}}^{U_{\sigma}^i} G(Q) dQ + U \int_{U_{\sigma}^n}^{\infty} G(Q) d(Q)$$

We can write

$$\begin{aligned} \int_{U_\sigma^{i-1}}^{U_\sigma^i} G(Q) dQ &= \int_0^{U_\sigma^i} G(Q) dQ - \int_0^{U_\sigma^{i-1}} G(Q) dQ \\ &= Q(U^i) - Q(0) - Q(U^{i-1}) + Q(0) \end{aligned}$$

Also

$$\begin{aligned} \int_{U_\sigma^n}^{\infty} G(Q) dQ &= \int_0^{\infty} G(Q) dQ - \int_0^{U_\sigma^n} G(Q) dQ \\ &= Q(\infty) - Q(0) - Q(U^n) + Q(0) \end{aligned}$$

$$\begin{aligned} f(Y) &= \sum_{i=1}^n F_i [Q(U^i) - Q(U^{i-1})] \\ &\quad + S_i [Q(U^i) - Q(U^{i-1})] \\ &\quad + Q[W(\infty) - Q(U^n)] \end{aligned}$$

$$\begin{aligned} f(Y) &= \sum_{i=1}^n [F_i + PS_i] [Q(U^i) - Q(U^{i-1})] \\ &\quad + U[Q(\infty) - Q(U^n)] \end{aligned}$$

So $f(y)$ can be written as

$$\begin{aligned}
 f(y) &= UQ(\infty) + \sum_{i=1}^n [F_i + PS_i] Q(U^i) - [F_i + PS_i] Q(U^{i-1}) - UQ(U^n) \\
 &\quad \text{let } F_i + PS_i = K_i \\
 f(y) &= UQ(\infty) + \sum_{i=1}^{n-1} [K_i - K_{i+1}] Q(U^i) - K_i Q(U^n) - UQ(U^n) \\
 &\quad \vdots \\
 f(y) &= UQ(\infty) + \sum_{i=1}^{n-1} [K_i - K_{i+1}] Q(U^i) - [U - K_n] Q(U^n) \\
 &\quad \text{Finally} \\
 f(y) &= U \int_0^\infty G(Q) dQ + \sum_{i=1}^{n-1} [K_i - K_{i+1}] \int_0^{U^i_\sigma} G(Q) dQ - [U - K_n] \int_0^{U^n_\sigma} G(Q) dQ
 \end{aligned}$$

Since $Q(x) = \int_0^x G(Q) dQ$ is a concave function of X because $G(Q)$ is monotonically decreasing (Chapter 3), and $F_i + PS_i \leq F_{i+1} + PS_{i+1}$ by merit order definition, I is convex function of U^i and of y . The function $f(Y)$ is just the expectation taken over all outage states which is a convex combination of convex function (Bazaraa and Shetty, 1979) since $\sum \Theta = 1$ and $\Theta \geq 0$ as stated in Chapter 3.

From KKT conditions and the convexity of $f(Y)$ we showed that Y minimizes $\gamma(Y, U, W)$ over $Y \geq 0$. Therefore minimizing $\gamma(Y, U, W)$ is equivalent to minimizing the function $f(Y) + \text{linear terms}$. A convex function plus added linear terms is also a convex function, so we verified the KKT optimality conditions.

An important result of the above is that the solution of the primal subproblem is equivalent to the solution of the dual subproblem. Due to this fact, we can replace the primal subproblem with its dual in the master problem. If both the primal and the dual are feasible, they have a finite solution with equal values.

The Master Problem

Recall the ECCP problem

$$\text{MINIMIZE}_{x,y} \sum_{t=1}^T \left[\sum_{ij} C_{ijt} X_{ijt} + \sum_{ij} F_{ijt} EG_{ijt}(y_t) + \sum_{ij} P_t S_{ijt} EG_{ijt}(y_t) \right]$$

Subject to

$$Y_{ijt} \leq \text{Cap}_{ijt} X_{ijt} \quad (1)$$

$$EUE_t(Y_{ijt}) \leq R_t \quad (2)$$

$$Y_{ijt} \geq m_{ijt} \text{Cap}_{ijt} \quad (3)$$

$$X_{ijt} = 0, 1 \quad (4)$$

Let X be the set of all 0-1 variables X_{ijt} that satisfy the above constraints.

Then, ECCP can be written as

$$\begin{aligned} \text{MINIMIZE}_x \min \sum_{t=1}^T \left[\sum_{ij} C_{ijt} X_{ijt} + \sum_{ij} F_{ijt} EG_{ijt}(y_t) + \sum_{ij} P_t S_{ijt} EG_{ijt}(y_t) \right] \\ \text{S.T.} \quad Y_{ijt} \leq \text{Cap}_{ijt} X_{ijt} \\ Y_{ijt} \geq m_{ijt} \text{Cap}_{ijt} \end{aligned}$$

which is same as

$$\begin{aligned} \text{MINIMIZE}_x \sum_{t=1}^T \sum_{ij} C_{ijt} X_{ijt} + \min \sum_{ij} \sum_{t=1}^T F_{ijt} EG_{ijt}(y_t) + \sum_{ij} \sum_{t=1}^T P_t S_{ijt} EG_{ijt}(y_t) \\ \text{S.T.} \quad Y_{ijt} \leq \text{Cap}_{ijt} X_{ijt} \\ Y_{ijt} \geq m_{ijt} \text{Cap}_{ijt} \end{aligned}$$

The optimization within the inner brackets is just the operating subproblem. As shown before the feasible solution of the primal subproblem is equivalent to the

feasible solution of the dual subproblem. So the inner optimization can be replaced by its dual.

$$\begin{aligned} & \text{maximum minimum } \{U_t[EUE_t(Y_t)-R_t] + W_t[Y_t-Cap_t X] + FEG_t(Y_t) + PSEG_t(Y_t)\} \\ & U_t, W_t \geq 0 \quad Y_t \geq 0 \end{aligned}$$

where W_t is a vector of the dual multipliers on the capacity constraint and U_t is a scalar multiplier on the unserved energy constraint.

Substituting the Lagrangian dual into ECCP yields a new form of ECCP:

$$\begin{aligned} & \text{minimize } \{CX + \max \min \{U_t[EUE_t(Y_t)-R_t] + W_t[Y_t-Cap_t X] + FEG_t(Y_t) + PSEG_t(Y_t)\} \\ & U_t, W_t \geq 0 \quad Y_t \geq 0 \end{aligned}$$

which is equivalent to

$$\begin{aligned} & \min \max \{ CX + \min \{U_t[EUE_t(Y_t)-R_t] + W_t[Y_t-Cap_t X] + FEG_t(Y_t) + PSEG_t(Y_t)\} \\ & U_t, W_t \geq 0 \quad Y_t \geq 0 \end{aligned}$$

Then the ECCP can be written

$$\text{ECCP2} = \quad \text{Minimize } Z$$

Subject to

$$\begin{aligned} CX + \sum_{Y_t \geq 0} \min \{U_t[EUE_t(Y_t)-R_t] + W_t[Y_t-Cap_t X] + FEG_t(Y_t) + PSEG_t(Y_t)\} & \leq Z \\ & \text{for all } U_t, W_t \geq 0 \\ & X \geq 0 \end{aligned}$$

The optimal value of this problem is equivalent to the optimal value of the original formulation.

Theorem (based on the Strong duality theorem in Bazaraa and Shetty, 1979)

- a) Problem ECCP2 has a feasible solution $\iff$ Problem ECCP has a feasible solution.
- b) Problem ECCP2 is feasible without having an optimal solution $\iff$ Problem ECCP is feasible without having an optimal solution.

Solution of the original problem has been reduced to solving Problem ECCP2 for (Z, X) and then solving the operating subproblem to obtain the optimal values of X . Unfortunately, solution of Problem ECCP2 is hindered by the fact that it has one constraint for each extreme point and this may be an enormous number in a problem of even moderate dimension. However, only a small fraction of the constraints will be binding at an optimal solution. This is a natural setting for applying the relaxation strategy (Lasdon, 1970). We begin with only a few (or no) constraints, and solve the resulting modified ECCP2, MECCP2. such that:

$$CX + \sum \min \{U_t[EUE_t(Y_t) - R_t] + W_t[Y_t - \text{Cap}_t X] + FEG_t(Y_t) + PS EG_t(Y_t^*)\} \leq Z$$

$$Y_t \geq 0$$

$$\text{for some } U_t, W_t \geq 0$$

A test is made to see if this solution satisfies the remaining constraints. If so, the solution is optimal, since the objective has been minimized over a set containing

constraints. If not, a constraint is added which is not satisfied by the current solution.

This violated constraint can be generated by solving the dual subproblem. For each

period t , assume that an initial feasible solution $x^1 \in X$ is known (start with $X_{ijt}^1 = 0$):

$$\text{maximum minimum } \{U_t[EUE_t(Y_t) - R_t] + W_t[Y_t - \text{Cap}_t X^1] + FEG_t(Y_t) + PSEG_t(Y_t)\}$$

$$U_t, W_t \geq 0 \quad Y_t \geq 0$$

As noted above, this problem is the dual of the operating subproblem

$$\text{minimize } \sum \sum FEG(Y) + \sum \sum PSEG(Y)$$

subject to

$$EUE_t(Y) \leq R_t$$

$$Y_t \leq \text{Cap}_t X^1$$

$$Y_t \geq m_t \text{Cap}_t$$

The optimal primal solution Y_t^* is obtained by solving this subproblem, and the dual multipliers U_t^* and W_t^* are determined from KKT conditions for this problem. As noted previously, the constraints generated by solving the subproblems are the most violated of the constraints not yet included in the master problem. Hence if the current trial solution (Z, X) satisfies the new constraints it generates, then it is optimal. Furthermore, the value of Z is a lower bound on the cost of the optimal solution, and if X^1 is feasible, the cost of this solution is an upper bound on the optimal cost.

$$UB = CX^1 + \sum FEG_t(Y_t^*) + \sum PSEG_t(Y_t^*)$$

These bounds can be used to terminate the procedure prior to optimality with known

error bounds. If a satisfactory solution has been found, then the newly generated constraints are added to the master problem, which is then re-solved to generate a new trial plan.

The constraint generated for the MECCP2 problem is

$$CX + \sum \min \{U_i^*[EUE_i(Y^*)-R_i] + W1_i^*[Y_i^*-Cap_iX] + FEG_i(Y^*) + PS EG_i(Y^*)\} \leq Z$$

From the complementary slackness condition we have

$$U_i^*[EUE_i(Y^*)-R_i] = 0$$

$$W1_i^*[Y_i^*-Cap_iX] = 0$$

$$\begin{aligned} W1_i^*[Y_i^*-Cap_iX] &= W1_i^*[Y_i^*-Cap_iX^*] + W1_i^*Cap_i(X^*-X) \\ &= W1_i^*Cap_i(X^*-X) \end{aligned}$$

Since Y_i^* solves the primal subproblems with optimal multipliers U_i^* , $W1_i^*$, Y_i^* solves the minimization problem above; therefore,

$$L(X, W1) = CX + \sum W1_i^*Cap_i(X^*-X) + FEG_i(Y^*) + PS EG_i(Y^*) \leq Z$$

Note that the above constraint does not contain the minimization function any more and can be easily evaluated. Since $W1 \geq 0$, $L(X, W1) \leq Z$ must be one of the constraints in ECCP2. Therefore, if we only include this constraint, we obtain the first relaxed master problem, RMECCP₁:

min Z

S.T.

$$CX + \sum W1_i^*Cap_i(X^*-X) + FEG_i(Y^*) + PS EG_i(Y^*) \leq Z$$

RMECCP₁ is a linear programming problem.

The solution to the above problem is straight forward:

$$Z = \sum CX + W1_t \text{Cap}_t(X1-X) + FEG_t(Y^{\wedge}_t) + PS EG_t(Y^{\wedge}_t)$$

Now let (X,Z) solve RMECCP₁. Since RMECCP₁ is a relaxed problem of ECCP2, and ECCP2 is equivalent to ECCP, therefore Z is a lower bound of ECCP, that is (LB= lower bound)

$$LB = Z = \sum CX + W1_t \text{Cap}_t(X^{\wedge}-X) + FEG_t(Y^{\wedge}_t) + PS EG_t(Y^{\wedge}_t)$$

If UB = LB, then (X,Z) is an optimal solution to ECPP. Otherwise, we solve the following primal subproblem:

$$\text{minimize } \sum \sum FEG(Y) + \sum \sum PSEG(Y)$$

subject to

$$EUE_t(Y) \leq R_t$$

$$Y_t \leq \text{Cap}_t X^{\wedge}$$

$$Y_t \geq 0$$

for period t. Also, we find the dual multipliers of the primal subproblem. Now if

$$CX + \sum FEG_t(Y^{\wedge}_t) + PS EG_t(Y^{\wedge}_t) \leq LB, \text{ we conclude that } (x,y) \text{ is an optimal}$$

solution to ECCP and we can stop here, otherwise, we evaluate L(X,W):

$$L(X,W) = CX + \sum W_t \text{Cap}_t(X^{\wedge}-X) + FEG_t(Y^{\wedge}_t) + PS EG_t(Y^{\wedge}_t)$$

Note that $L(X,W) = CX + \sum FEG_t(Y^{\wedge}_t) + PS EG_t(Y^{\wedge}_t) > Z$. Since $W \geq 0$, therefore

$L(X,W) \leq Z$ is one of the constraints and is violated by current solution (X,Z).

Now we add the constraint $L(X,W) \leq Z$ to RMECCP₁ and obtain the second relaxed problem of ECCP, RMECCP₂

min Z

S.T

$$CX + \sum W1Cap(X1-X) + F EG(Y1) + PS EG(Y1) \leq Z$$

$$CX + \sum W1Cap(X2-X) + F EG(Y2) + PS EG(Y2) \leq Z$$

RMECCP₂ is a linear programming problem. If UB= LB stop. Otherwise, a new subproblem for period t is created and solved again.

The resulting algorithm involves iteration between two problems. The first is the problem MECCP2 in the variables (z,y) to which constraints are successively added. The second is the dual subproblem which tests the optimality of a solution to MECCP2 and, if necessary, provides a new constraint.

Finally, the relaxed master problem can be written

Minimize Z

Subject to

$$\sum \sum CX + \sum \sum W_t^k Cap_t(X^k - X) + \sum \sum F EG_t(Y_t^k) + \sum \sum PS EG_t(Y_t^k) \leq Z \quad k$$

$$= 1, \dots, K$$

$$X \geq 0$$

where X^k is the k^{th} trial solution generated and Y_t^k and W_t^k are the associated primal and dual solutions to the operating subproblems. This master problem is a linear program and it is equivalent to the original ECCP problem.

Next, to calculate the optimal objective value, we will use the cumulant

method (Stremel et al., 1980), which we will discuss in Chapter 5 along with the derivative of the probabilistic simulation function.

Conclusion

In Chapter 4, we derived the operating subproblem from the master problem by fixing the X values (the zero-one decision variables). We found the dual operating problem and used the KKT optimality conditions to find the dual multipliers U and W . The U 's are associated with the unserved energy constraints and the W 's are associated with the capacity constraints. Finally, we reformulated the master problem as a linear programming model, and we found an iterative procedure to solve the ECCP problem.

CHAPTER 5

EXPECTED ENERGY PRODUCTION COST FUNCTION AND ITS DERIVATIVES BY THE METHOD OF CUMULANTS

In this chapter we will outline the Cumulant method (Stremel et al., 1980) of calculating the expected energy generation in the generating system by probabilistic simulation. In addition, we will present a closed-form and a computationally efficient expression for obtaining the derivatives, U , W , based on the cumulant method. In Chapter 4, the optimal solution to the operating subproblem of the probabilistic model and its associated dual multipliers were derived. The actual computation of the dual multipliers will be discussed in this chapter.

The Cumulant Method

In this section we will introduce a production costing algorithm for a power plant which uses an analytical approximation. This algorithm will expedite the calculation of expected energy generation by using cumulants to represent the Equivalent Load Curve (ELC). The analytical approximation allows for the introduction of increased speed and accuracy. However, this approximation has been found to be marked for small generation systems or systems with few generating

units, and for systems with forced outage rates (Levy and Kahn, 1982).

In the cumulant method, we perform the convolution of unit outages which is a probability distribution with the load curve distribution in the merit order of loading. It has been shown (Rau and Schenk, 1979) that a successive convolution of several density functions can be expressed by the Gram-Charlier series which can represent the Equivalent Load Curve (ELC)

$$ELC_r(Z_1) = 1 - \int_{-\infty}^{Z_1} N(Z) dZ + \frac{G_1}{3!} N^2(Z_1) - \frac{G_2}{4!} N^3(Z_1) - \frac{10}{6!} G_1^2 N^5(Z_1)$$

Having obtained the convolution, the energy calculation, that is the area under the ELC, is obtained by a mere look-up of the normal probability table.

In the cumulant method, the various random variables are described by their cumulants. Cumulants are linear combinations of statistical moments. In case of a two-state representation of the power unit, the i^{th} unit in a system, the failure probability density function PDF consists of two impulses, one of magnitude p_i which is the availability rate and one of magnitude q_i which is the failure or outage rate. The moments of such two-states failure PDFs are easy to obtain. For example, the first moment of the outage of unit 1 is equal to the multiplication of the capacity of that unit times the outage rate (q_i). According to Stremal et al., (1980), the convolution of different unit outages is nothing but a process of adding the individual cumulants according to the indicated procedure (see next section) and the deconvolution is a process of subtracting the cumulants. This procedure is used to obtain the capacity

outage table of a generating system.

As a result of this representation, convolution is accomplished by cumulant addition and deconvolution is accomplished by the cumulant subtraction. The cumulants of the equivalent load are then used in the Gram-Charlier series to approximate the associated equivalent load duration curve. In the application of the cumulant method, the number of cumulants used to describe each random variable has been the subject of extensive study (Jenkins, 1977; Rau et al., 1979; Schenk, 1977; Stremel et al., 1979; Stremel et al., 1980). More cumulants result in higher computational requirements, but do not always improve the relative accuracy of the simulation results (Lin et al., 1989). This due to the fact that the Gram-Charlier series is not uniformly convergent. Four cumulants were used in the simulation reported in this theses.

The load duration probability distribution can also be described by a set of cumulants. Consequently, just as the generation outage cumulants can be used to construct the capacity outage table, the sum of the outage and load cumulant can be used to construct the equivalent load curve.

The Formulas of the Cumulant Method

The essential details of this method are extracted from Stremel et al., (1980) and are shown below.

Let

LK_k be the k th cumulant of the hourly load distribution (which can also describe the load duration curve)

$GK_k(r)$ be the k th cumulant of generation outage distribution after the r th unit has been convolved

g_{ki} be the k th cumulant of the i th generation unit

p_i be the probability the i th generation unit is available

q_i be the outage or failure rate of the i th generation unit

L_t be the hourly load at time t

M_{ki} be the k th moment of the outage distribution of the i th unit

$$= \sum p_i q_i^k$$

LM_k be the k th cumulant of the hourly load distribution

ELC Equivalent Load Curve

$ELC_k(r)$ be the k th cumulant of the ELC after r units have been convolved

$j = (1, \dots, m)$, number of outage states

$i = (1, \dots, n)$, number of units or capacity blocks

$k = (1, 2, 3, 4)$, number of cumulants

$t = (1, \dots, T)$, number of hours in period

The cumulants for each unit which will define the generation outage distribution for the system are:

$$g_{1i} = M_{1i}$$

$$g_{2i} = M_{2i} - M_{1i}^2$$

$$g_{3i} = M_{3i} - 3M_{2i}M_{1i} + 2M_{1i}^3$$

$$g_{4i} = M_{4i} + 6M_{2i}M_{1i}^2 - 4M_{3i}M_{1i} - 3M_{1i}^4 - 3(g_{2i})^2$$

Similarly the cumulants of the hourly load distribution are:

$$LM_j = 1/T \sum L_t^j \text{ for } j=1,2,3,4$$

$$LK_1 = LM_1$$

$$LK_2 = LM_2 - LM_1^2$$

$$LK_3 = LM_3 - 3LM_2LM_1 + 2LM_1^3$$

$$LK_4 = LM_4 + 6LM_2 LM_1^2 - 4LM_3LM_1 - 3LM_1^4 - 3LK_2^2$$

The above calculations are an overhead investment since they will not be revised unless the basic outage data or the loads change. What follows are the necessary calculations to perform the Gram-Charlier approximation of the equivalent load curve (Stremel et al., 1980).

The cumulants for the equivalent load curve after the first r units have been convolved on the system are

$$\begin{aligned} ELK_k(r) &= LK_k + GK_k(r) \\ &= Lk_k + \sum g_{ki} \quad \text{for } k=1,2,3,4 \end{aligned}$$

From the Gram-Charlier series we can represent the Equivalent Load Curve (Rau and Schenk, 1974; Schenk, 1977) as

$$ELC_r(Z_1) = 1 - \int_{-\infty}^{Z_1} N(Z) dZ + \frac{G_1}{3!} N^2(Z_1) - \frac{G_2}{4!} N^3(Z_1) - \frac{10}{6!} G_1^2 N^5(Z_1)$$

where the normal probability density function $N(Z)$ and its derivatives are given by

$$N(Z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{Z^2}{2}\right)$$

$$Z_1 = \frac{X - \mu}{\sigma}$$

$$N^r(Z) = \frac{d^r}{dZ^r} N(Z); \quad r=1, 2, \dots$$

where

X is the equivalent load level for which we need to know the value of the equivalent load duration curve, i.e. the capacity of the r units

μ is the mean = $ELK_1(r)$

σ is the variance = $[ELK_2(r)]^{0.5}$

Z_1 is a standardized normal variable

$N(Z)$ is the normal distribution function evaluated at Z

The derivatives of the Normal distribution function are:

$$N^1(Z_1) = -Z_1 N(Z_1)$$

$$N^2(Z_1) = (Z_1^2 - 1) N(Z_1)$$

$$N^3(Z_1) = (-Z_1^3 - 3Z_1) N(Z_1)$$

$$N^4(Z_1) = (Z_1^4 - 6Z_1^2 + 3) N(Z_1)$$

$$N^5(Z_1) = (-Z_1^5 + 10Z_1^3 - 15Z_1) N(Z_1)$$

and the coefficients of Gram-Charlier series expansion are:

$$G_1 = 1/\sigma^3 ELK_3(r)$$

$$G_2 = 1/\sigma^4 ELK_4(r)$$

This procedure allows for the determination of the ELC at a position in the loading order without having to numerically convolve the previous units. The first requirement of this method is the calculation of cumulants of the load of each different type of generating unit. These calculations are independent of each other and can be done in advance. An example will clarify the method.

Example

The mean of Gainesville Regional Utilities system load is 171 MW and the standard deviation is 50.3 MW. The load cumulants for a year's hourly loads have been determined and summarized in table below.

Table 5.1 Load Cumulants

Mean	$\mu = \text{LK}_1$	171
Variance	$\sigma^2 = \text{LK}_2$	2526.7
Skewness	LK_3	99157.8
Kurtosis	LK_4	22546262

The first three generation units in the loading order (merit order) are 11 MW, 235 MW, and 44 MW respectively. The forced outage level of each unit is 27.6%, 13% and 15%. What is the expected generation of the fourth unit which is 81 MW with an availability of 87.1%?

The outage cumulants for the four units are listed in the table below.

Table 5.2 The Outage Cumulants

	g_{1i}	g_{2i}	g_{3i}	g_{4i}
Unit 1	3.04	24.17	119.15	-582.00
Unit 2	30.55	6245.90	1086170.00	110861290.00
Unit 3	6.60	246.84	7602.67	112302.00
Unit 4	10.50	737.18	44306.44	1576015.00
Total	51.69	7256.09	1138201.26	112549029.00

The moments for each unit of the system are:

$$M_{11} = 11 * 0.276 = 3.04$$

$$M_{21} = 11^2 * 0.276 = 33.4$$

$$M_{31} = 11^3 * 0.276 = 367.4$$

$$M_{41} = 11^4 * 0.276 = 4041$$

The cumulants for each unit which will determine the generation outage distribution for the system are:

$$g_{11} = M_{11} = 3.04$$

$$g_{21} = 33.4 - (3.04)^2 = 24.17$$

$$g_{31} = 367.4 - 3(33.4)3.04 + 2(3.04)^3 = 119.15$$

$$g_{41} = 4041 + 6(33.4)(3.04)^2 - 4(367.4)(3.04) - 3(3.04)^4 - 3(24.17)^2 = -582$$

The Gram-Charlier parameters for the ELC facing the fourth unit in the loading order are:

$$ELK_1(3) = M = 171 + 30.55 + 3.04 + 6.60 = 211.19$$

$$ELK_2(3) = \sigma^2 = 2,526.7 + 24.18 + 6245.9 = 9,042$$

$$ELK_3(3) = 999,157.8 + 119.15 + 1086170 + 7602.67 = 1,193,049.36$$

$$ELK_4(3) = 22,546,262 - 582 + 110861290 + 112302 = 133,519,272$$

The coefficients of the Gram-Charlier expansion are:

$$G_1 = 1,237,355/(9,780)^{3/2} = 1.279$$

$$G_2 = 135,095,287/(9,780)^2 = 1.412$$

Now that the coefficients of the approximation to the equivalent load curve have been determined, we need to evaluate the approximation at $IC_3 = 11 + 235 + 44 = 290$ and $IC_4 = 11 + 235 + 44 + 81 = 371$.

For IC_3 :

$$Z = 290 - 221.69 / (9,042)^{1/2} = 0.82$$

$$N(Z) = 0.2841 \text{ (Table lookup for Standard Normal)}$$

$$\int N(X) Dx = 0.1841 \text{ (Table lookup for Standard Normal)}$$

The derivatives are

$$N^{(2)}(0.82) = ((0.82)^2 - 1) 0.2841 = -0.0911$$

$$N^{(3)}(0.82) = 0.5433$$

$$N^{(5)}(0.82) = -2.029$$

The approximation to the Equivalent Load Curve at the point 290 is

$$\begin{aligned} \text{ELC}_3(290) &= 1 - 0.1841 + 1.279 (-0.0911)/6 - 1.412 (0.5433)/24 \\ &\quad - 10 (1.279)^2(-2.029)/720 \\ &= 0.8121 \end{aligned}$$

The second point is IC_4 which is equal to 371

For IC_4 ,

$$Z = 371 - 221.69 / (9,780)^{1/2} = 1.51$$

$$N(Z) = 0.1284 \text{ (Table lookup for Standard Normal)}$$

$$\int N(X) Dx = 0.0548 \text{ (Table lookup for Standard Normal)}$$

The derivatives are

$$N^{(2)}(1.51) = 0.1627$$

$$N^{(3)}(1.51) = 0.1416$$

$$N^{(5)}(1.51) = 0.4898$$

The approximation to the Equivalent Load Curve at the point 290 is

$$\begin{aligned} \text{ELC}_4(290) &= 1 - 0.0548 + 1.279 (0.1627)/6 - 1.412 (0.1416)/24 \\ &\quad - 10 (1.279)^2(0.4898)/720 \\ &= 0.9604 \end{aligned}$$

Therefore, the expected generation of the fourth unit is

$$EG_4 = 0.871 * 81 * (0.8121 + .9604)/2 = 62.53 \text{ MW}$$

A spreadsheet program has been developed to calculate the expected generation of each unit. A copy of that program is listed in Appendix A.

Advantages of the Cumulant Method

The cumulant method offers quick evaluations for production costing without compromising accuracy. As it can be seen from the example above, there are many advantage of using the cumulant methods. these advantages can be summarized as follows:

- 1- The individual plant cumulants and load cumulants are calculated one time only and used repeatedly throughout the analysis.
- 2- The determination of the coefficients of the Gram-Charlier approximation for the Equivalent Load Curve for the jth unit involve simple addition and manipulation of load and plant cumulants.
- 3- The solution of the Gram-Charlier approximation involves one table look up and evaluation of the three derivative terms.
- 4- Evaluation of the ELC at IC_{j-1} and IC_j allows for an estimate of the expected generation.

The Derivatives of the Objective Function

The derivatives of the production costing simulation provide valuable guidance in solution of electric system planning models. For each set of decisions (i.e. capacity changes), the derivatives of the objective function with respect to the decision options (capacity changes) show a way to change the decision that will reduce/increase the value of the objective function. We will present a closed-form expression for the derivatives based on the cumulant implementation of the equivalent load methodology that was explained in the previous section.

The derivatives we desire are $\partial EG_j / \partial Cap_i$, the change in energy served by unit j with respect to a change in the capacity of the i th resource. The energy served by unit j can be expressed by the following equation:

$$EG_j = 1/2 * P_j * Cap_j (ELC_{j-1} + ELC_j)$$

where ELC_i is the equivalent load duration after dispatching the i th resource.

Differentiating the above with respect to Cap_i yields

$$\partial EG_j / \partial Cap_j = 1/2 * P_j * Cap_j (\partial ELC_{j-1} / \partial Cap_i + \partial ELC_j / \partial Cap_i) + 1/2 * P_j (ELC_{j-1} + ELC_j)$$

Thus the task now is finding $\partial ELC_j / \partial Cap_i$. Calculating any other derivative of interest uses the same approach (i.e. $\partial EUE_j / \partial R_j$). The variable of interest replaces the capacity Cap_i .

The first step in finding this derivative is finding the derivative of the ELC with respect to the cumulants. The ELC is given by the following equation from the previous section.

$$ELC_j = 1 - \int_{-\infty}^{Z_j} N(Z) dZ + \frac{G_{1j}}{3!} N^2(Z_j) - \frac{G_{2j}}{4!} N^3(Z_j) - \frac{10}{6!} G_{1j}^2 N^5(Z_j)$$

Now let us calculate $\partial ELC_j / \partial ELK_{sj}$.

$$\begin{aligned} \frac{\partial ELC_j}{\partial ELK_{sj}} &= \frac{\partial}{\partial ELK_{sj}} \int_{Z_j}^{\infty} N(W) dW + \frac{\partial G_{1j}}{\partial ELK_{sj}} \frac{N^2(Z_j)}{6} + \frac{G_{1j}}{6} \frac{\partial N^2(Z_j)}{\partial ELK_{sj}} \\ &- \frac{\partial G_{2j}}{\partial ELK_{sj}} \frac{N^3(Z_j)}{24} - \frac{G_{2j}}{24} \frac{\partial N^3(Z_j)}{\partial ELK_{sj}} - \frac{\partial G_{1j}^2}{\partial ELK_{sj}} \frac{N^5(Z_j)}{72} - \frac{G_{1j}^2}{72} \frac{\partial N^5(Z_j)}{\partial ELK_{sj}} \end{aligned}$$

which can be written as

$$\begin{aligned} \frac{\partial ELC_j}{\partial ELK_{sj}} &= \frac{\partial Z_j}{\partial ELK_{sj}} N(Z_j) + \frac{\partial G_{1j}}{\partial ELK_{sj}} \frac{N^2(Z_j)}{6} + \frac{G_{1j}}{6} \frac{\partial N^2(Z_j)}{\partial Z_j} \frac{\partial Z_j}{\partial ELK_{sj}} \\ &- \frac{\partial G_{2j}}{\partial ELK_{sj}} \frac{N^3(Z_j)}{24} - \frac{G_{2j}}{24} \frac{\partial N^3(Z_j)}{\partial Z_j} \frac{\partial Z_j}{\partial ELK_{sj}} - \frac{\partial G_{1j}^2}{\partial ELK_{sj}} \frac{N^5(Z_j)}{72} - \frac{G_{1j}^2}{72} \frac{\partial N^5(Z_j)}{\partial Z_j} \frac{\partial Z_j}{\partial ELK_{sj}} \end{aligned}$$

Or

$$\begin{aligned} \frac{\partial EL C_j}{\partial EL K_{sj}} &= \frac{\partial Z_j}{\partial EL K_{sj}} N(Z_j) + \frac{\partial G_{1j}}{\partial EL K_{sj}} \frac{N^2(Z_j)}{6} + \frac{G_{1j}}{6} N^3(Z_j) \frac{\partial Z_j}{\partial EL K_{sj}} \\ &- \frac{\partial G_{2j}}{\partial EL K_{sj}} \frac{N^3(Z_j)}{24} - \frac{G_{2j}}{24} N^4(Z_j) \frac{\partial Z_j}{\partial EL K_{sj}} - \frac{\partial G_{1j}^2}{\partial EL K_{sj}} \frac{N^5(Z_j)}{72} - \frac{G_{1j}^2}{72} N^6(Z_j) \frac{\partial Z_j}{\partial EL K_{sj}} \end{aligned}$$

For s=1

$$\frac{\partial Z_j}{\partial EL K_{1j}} = -\frac{1}{\sigma_j}$$

Since

$$Z_j = \frac{X - EL K_{1j}}{\sigma_j}$$

$$\frac{\partial G_{ij}}{\partial EL K_{1j}} = 0 \quad \text{for } i=1, 2, 3$$

Since

$$G_{ij} = \frac{EL K_{3j}}{\sigma_j^3}$$

$$\frac{\partial EL C_j}{\partial EL K_{1j}} = -\frac{1}{\sigma_j} N(Z_j) - \frac{G_{1j}}{6} N^3(Z_j) \frac{1}{\sigma_j} + \frac{G_{2j}}{24} N^4(Z_j) \frac{1}{\sigma_j} + \frac{G_{1j}^2}{72} N^6(Z_j) \frac{1}{\sigma_j}$$

For $s=2$

where

$$\frac{\partial Z_j}{\partial ELK_{2j}} = -\frac{Z_j}{2\sigma_j^2}$$

Since

$$Z_j = \frac{X - ELK_{1j}}{\sigma_j} = \frac{X - ELK_{1j}}{ELK_{2j}^{0.5}}$$

$$\frac{\partial G_{ij}}{\partial ELK_{2j}} = \frac{-(i+2) G_{ij}}{2\sigma_j^2} \quad \text{for } i=1, 2, 3$$

Since

$$G_{ij} = \frac{ELK_{(i+2)j}}{\sigma_j^{(i+2)}} = \frac{ELK_{(i+2)j}}{ELK_{2j}^{(i+2)/2}}$$

$$\begin{aligned} \frac{\partial ELC_j}{\partial ELK_{2j}} &= \frac{-Z_j}{2\sigma_j^2} N(Z_j) - \frac{3G_{1j}}{2\sigma_j^2} \frac{N^2(Z_j)}{6} - \frac{G_{1j}}{6} N^3(Z_j) \frac{Z_j}{2\sigma_j^2} \\ &+ \frac{4G_{2j}}{2\sigma_j^2} \frac{N^3(Z_j)}{24} + \frac{G_{2j}}{24} N^4(Z_j) \frac{Z_j}{2\sigma_j^2} + \frac{3G_{1j}^2}{\sigma_j^2} \frac{N^5(Z_j)}{72} + \frac{G_{1j}^2}{72} N^6(Z_j) \frac{Z_j}{2\sigma_j^2} \end{aligned}$$

For $s=3$

$$\frac{\partial Z_j}{\partial ELK_{3j}} = 0$$

Since

$$Z_j = \frac{X - ELK_{1j}}{\sigma_j} = \frac{X - ELK_{1j}}{ELK_{2j}^{0.5}}$$

$$\frac{\partial G_{1j}}{\partial ELK_{3j}} = \frac{1}{\sigma_j^3}$$

Since

$$G_{1j} = \frac{ELK_{3j}}{\sigma_j^3}$$

The partial derivative of Z_j and G_{1j} with respect to $ELK_{>3j}$ is zero.

$$\frac{\partial ELC_j}{\partial ELK_{3j}} = \frac{1}{\sigma_j^2} \frac{N^2(Z_j)}{6}$$

The actual served energy is given as follows:

$$EG_j = T * 1/2 * P_j * Cap_j (ELC_{j-1} + ELC_j)$$

Where T is the number of hours in period over which the production simulation is being carried out.

Differentiating EG_j with respect to capacity we obtain

$$\begin{aligned} \partial EG_j / \partial Cap_j &= T * 1/2 * P_j * Cap_j (\partial ELC_{j-1} / \partial Cap_j + \partial ELC_j / \partial Cap_j) \\ &+ T * 1/2 * P_j (ELC_{j-1} + ELC_j) \end{aligned}$$

which can be written as

$$\begin{aligned} \frac{\partial EG_j}{\partial Cap_i} = & T * 1/2 * P_j \left[\sum_{s=1}^3 \left[\frac{\partial ELC_j}{\partial ELK_{sj}} \frac{\partial ELK_{sj}}{\partial Cap_i} + \frac{\partial ELC_{j-1}}{\partial ELK_{sj}} \frac{\partial ELK_{sj}}{\partial Cap_i} \right] \right. \\ & \left. + T * 1/2 * P_i (ELC_{j-1} + ELC_j) \right] \end{aligned}$$

Several of the partial derivatives remain to be calculated. Let k_1^i and k_2^i be the first and second outage cumulants, respectively, of one unit of resource i (Fancher et al., 1986). Then

$$\begin{aligned} \frac{\partial ELK_{sj}}{\partial Cap_i} &= uk_s^i & \text{for } j \geq i \\ &= 0 & \text{for } j < i \end{aligned}$$

where

$$uk_s^i = \frac{k_s^i}{S_i}$$

k_s^i = outage cumulant for unit i
 S_i = size of unit i

Result 1 for $i < j$, If unit i is utilized before unit j , then

$$\partial EG_j / \partial Cap_i = 0$$

Result 2 for $i = j$

$$\begin{aligned} \partial EG_j / \partial Cap_j = & 1/2 * P_j * Cap_j (\partial ELC_{j-1} / \partial Cap_i + \partial ELC_j / \partial Cap_i) + 1/2 * P_j (ELC_{j-1} \\ & + ELC_j) \end{aligned}$$

Result 3 for $i > j$, If unit i is utilized after unit j , then

$$\begin{aligned} \partial EG_j / \partial Cap_i = & 1/2 * P_j * Cap_j (\partial ELC_{j-1} / \partial Cap_i + \partial ELC_j / \partial Cap_i) + 1/2 * P_j (ELC_{j-1} \\ & + ELC_j) \end{aligned}$$

Examples

We will use the example in the previous section to show the derivative of each unit with respect to its capacity. Table 5.3 presents the derivative of the expected generation of unit 1,2,3 and 4 with respect to the capacity of unit 1.

Table 5.3 Derivatives With Respect to the Capacity of Unit 1

Unit Number	CAPACITY (MW)	Mwh	$\partial EG_j / \partial \text{Cap}_1$ (MWh/MW)
1	11	66400	2885
2	235	1343765	355
3	44	218678	110
4	81	500297	77

Table 5.4 presents the derivative of the expected generation of units 1,2,3 and 4 with respect to the capacity of unit 2. Of course, the derivative of the expected generation of unit 1 with respect to the capacity of unit 2 is zero.

Table 5.4 Derivatives With Respect to the Capacity of Unit 2

Unit Number	CAPACITY (MW)	MWh	$\partial EG_j / \partial \text{Cap}_2$
1	11	63515	0
2	236	1350024	6614
3	44	218824	256
4	81	500409	189

This section shows the derivatives in which are used in the Emission Capacity Control Problem (ECCP) that was formulated in Chapter 3. Suppose that unit 2, in the example above, consists nuclear plant with a variable cost of \$10/MWh; unit 2 has coal-fired plant with a variable cost of \$20/MWh; unit 3 has an oil-fired plant with a variable cost of \$50/MWh; and unit 4 has a variable cost off \$55/MWh. Table 5.5 shows how multiplying the derivatives in table 5.3 by these variable costs and summing them yields the change in system operating cost that would result from having one additional megawatt of unit 2.

Table 5.5 Change in System Cost

Unit Number	$\partial EG_j / \partial Cap_2$	Variable Cost (\$/MWh)	Change in the system Cost (\$)
1	0	10	0
2	6614	20	132,280
3	256	50	12,800
4	189	55	10,395
Total			\$155,475

CHAPTER 6

COMPUTATIONAL RESULTS FOR EXAMPLE UTILITY SYSTEMS

This chapter discusses an implementation of the ECCP model presented in Chapter 4, and presents the results of three different runs using data based on the Gainesville Regional Utilities system and a synthetic utility system, called Scaled Down EPRI System D and E. The algorithm that was developed in Chapter 4 was programmed in a spreadsheet using the cumulant method to evaluate the production costing function. The Lagrangian multiplier calculations were also programmed in a spread sheet. The spreadsheet has the capacity to calculate the expected energy generated and the lagrangian multipliers for more than 100 power plants at one time. Three test problems were run using data based on the Gainesville Regional Utilities system and EPRI. The results of these runs indicate that the algorithm can indeed produce a sequence of trial solutions which get successively closer to an optimal feasible solution.

Outline of the Solution Procedure

The solution procedure of the Emission Capacity Controls Problem can be summarized as follows:

Initialization

1) Start with the initial solution $X_{10k}=0$

2) Construct the primal subproblem for each period and solve it to

obtain y^1_{ijt} and the Lagrangian multiplier W^1_{ijt}

If the subproblem is infeasible (the reliability constraint is violated),

check the expansion options and choose the least cost option.

3) Form the first relaxed master problem

Minimize Z

Subject to

$$\sum \sum C_{ijt} X_{ijt} + \sum \sum \{W^k_{ijt} \text{Cap}_{ijt}(X_{ijt}^k - X_{ijt}) + F_{ijt} \text{EG}(Y^k_{ijt}) + P_t S_{ijt} \text{EG}(Y^k_{ijt})\} \leq Z \quad k = 1, \dots,$$

K

$$X \geq 0$$

4) Obtain an upper bound

$$UB = \sum \sum CX + \sum \sum FEG(Y) + \sum \sum SPEG(Y)$$

Step 2

Solve the relaxed master problem and obtain (X^{k+1}, Z^k)

Set the lower bound to $LB = Z^k$

Does the subproblem converge? $UB = LB$?

a) yes Stop

b) no goto step 3

Step 3

For each period $t=1,2,\dots,T$

Form the subproblem

for each plant $i=1,\dots,I$, and

for each $j=1,\dots,J$

Compute W_{ijt}

If $UB > CX^k + \sum \sum FEG(y^k) + \sum \sum SPEG(y^k)$ then

$$UB = CX^k + \sum \sum FEG(y^k) + \sum \sum SPEG(y^k)$$

Else

Stop (X^k, y^k) is an optimal solution

Form relaxed problem, goto step 2

Gainesville Regional Utilities System

The Gainesville Regional Utilities (GRU) system consists of 10 units of three distinct types. The total system capacity is 478 MW. Of this amount, 49% is coal-fired (DH2), 2.3% is nuclear (CR3) and 48.7% is natural gas (JRK7, JRK8, KCT1, KCT2, KCT3, DH1, and DHCT1,2). In table 6.1, we list the unit number and name, fuel types, standard size (MW), forced outage (%), the operating cost of each plant (\$/MWh) and the emission rate of each unit (lb/MWh). The operating costs ranges from \$5/MWh for the nuclear power plant to \$9/MWh for the gas fired power plant. The emissions rate vary from "no emissions" of SO_2 for both the nuclear and gas power units to 9.8lb/MWh for the DH2 unit (coal).

Table 6.1 Plant Data

Plant Number and Name	Fuel type	Standard Size (MW)	Forced Outage (%)	Operating Cost (\$/MWh)	SO ₂ Emission Rate (lb/MWh)
1- CR3	Nuclear	11	27.6	5	0
2- DH2	Coal	235	13	19	9.8
3- JRK8	Gas	44	15	38.3	0
4- DH1	Gas	81	12.9	38.5	0
5- JRK7	Gas	22	4.9	42	0
6- DHCT1	Gas	20	6.6	60	0
7- DHCT2	Gas	20	6.3	70	0
8- KCT1	Gas	15	16.6	80	0
9- KCT2	Gas	15	17.5	70	0
10- KCT3	Gas	15	12.4	95	0
Total Capacity		478			

Load Data

The mean of the system load is 171 MW and the standard deviation is 50.3 MW. The planning horizon for our analysis is 10 years. The load cumulants for a year's hourly loads have been determined (see Appendix B) and summarized in table 6.2.

Table 6.2 Load Cumulants

Mean	LK1	171
Variance	LK2	2526.7
Skewness	LK3	99157.8
Kurtosis	LK4	22546262

For the initial year of the planning horizon the energy demand is 1,497,960 MWh per year. The load factor for the planning horizon is about 50%. The forecasted load growth is about 3% a year.

Retrofit Data

The Deerhaven coal-fired (DH2) unit is the only unit considered for an emissions control retrofit option since it is the only unit that emits SO_2 . We considered eight different options for that unit: 95% Flue Gas Desulfurization (FGD95), 50% Flue Gas Desulfurization (FGD50), switching to lower sulfur coal (LS), three types of dry limestone injection systems (DLI1, DLI2, and DLI3), limestone injection multistage burners (LIMB), and energy conservation (DSM). These options along with the percentage of SO_2 removal, the percentage of capacity penalty, and the increase in the operating cost are summarized in table 6.3.

Table 6.4 summarizes the new features of the Deerhaven (DH2) unit due to the retrofit options. In that table we show the new capacity of the unit as a result of a retrofit option, the new operating cost of unit 2 with option j ($j=1, \dots, 7$), the new emission rate, and the fixed cost of each retrofit option.

The effect of installing the FGD95 on unit 2 can be seen in the above table. While the emission rate is reduced by 95% (from 9.8 lb/MWh to 0.49 lb/MWh) the operating cost is increased by \$5/MWh (from \$19/MWh to \$24/MWh). In addition, there is an initial investment cost of about \$833,543. Another penalty imposed on the system is the reduction in the capacity of the unit from 236MW to 226MW as a result of installing FGD95.

Table 6.3 Retrofit Options

Option Number	Unit	Type	SO ₂ Removal (%)	Capacity Penalty (%)	Increase in Operating Cost (\$/MWh)
1	2	FGD	95	1	5
2	2	FGD	50	2	4
3	2	LS	50	1	2
4	2	DLI1	85	4	7
6	2	DLI2	70	4	5
6	2	DLI3	70	4	4
7	2	LIMB	50	4	7
Energy Conservation Option					
8	-----	DSM	Depends on the MW reduction	None	1

Table 6.4 The New Characteristics of Power Unit

Option Number	Plant Number	Retrofit type	New Capacity (MW)	Operating Cost (\$/MWh)	SO ₂ Emission Rate (lb/MWh)	Fixed Cost (\$)
1	2	FGD95	226	24	0.49	833,543
2	2	FGD50	231	22	4.9	500,126
3	2	FS	233	21	4.9	59,896
4	2	DLI1	226	26	1.47	598,190
6	2	DLI2	226	24	2.94	480,513
6	2	DLI3	226	24	2.94	166,709
7	2	LIMB	226	23	4.9	190,898
Energy Conservation Option						
8	-----	DSM	-----	Each unit will have an increased in its operating cost by \$1/MWh	-----	-----

Expansion Data

We considered four different options for building a new power plant. We will consider each option in every period of the time horizon of our analysis. Table 6.5 shows the plant type that is under consideration to be built, the size of the proposed plant, the forced outage, the operating cost in \$/MWh, the emission rate (lb/MWh), and the cost of building the proposed unit.

Table 6.5 Expansion Data

Plant type	Size (MW)	Forced Outage (%)	Operating Cost (\$/MWh)	Emission Rate (lb/MWh)	Cost (\$)
Gas Turbine	80	15	35	0	2,240,000
Gas Turbine	100	15	30	0	2,800,000
Coal	100	10	15	13	7,600,000

ECCP Formulation

In Chapter 3 we formulated the ECCP as a nonlinear programming model. In this section we will explicitly model and simulate the GRU system to find the minimum cost of supplying electricity and meeting the energy demand. CR3, DH2 and JRK8 must run at least 50% of its utilization level. For the initial year of the planning horizon the energy demand is 1,497,960 MWh per year. The load factor for the planning horizon is about 50%. The forecasted load growth is about 3% a year.

Recall the ECCP formulation from Chapter 3.

$$\begin{aligned} \text{MINIMIZE}_{x,y} \sum_{t=1}^T [& \sum_i \text{CNP}_{ijt} \text{NP}_{ijt} + \sum_{ij} \text{C}_{ijt} \text{X}_{ijt} + \sum_{ij} \text{F}_{ijt} \text{EG}_{ijt}(y_t) \\ & + \sum_{ij} \text{P}_t \text{S}_{ijt} \text{EG}_{ijt}(y_t)] \end{aligned}$$

$$Y_{ijt} \leq \text{Cap}_{ijt} X_{ijt} \quad (1)$$

$$\text{EUE}_t(Y_{ijt}) \leq R_t \quad (3)$$

$$Y_{ijt} \geq m_{ijt} \text{Cap}_{ijt} \quad (4)$$

$$X_{ijt} = 0, 1 \quad (5)$$

The Least Costs for GRU

In this section, we apply the solution procedure that we described in Chapter 4 to find the optimal expansion and least costly emission plan for GRU (see appendix B). Since the prices of emission allowances are uncertain, we will construct different scenarios for particular prices of allowance.

We will start with a price of \$150 for each emission allowance. Then, we will try different prices for allowances to see the affect of the changes in the price of the emission allowances. Considering a retrofit on Deerhaven power plant (DH2) and three different expansion options, the least cost was found by applying the partitioning solution procedure that was developed in Chapter 4. The findings are summarized in Table 6.6 and shown in Figure 1.

Table 6.6 GRU Least Cost Plan

	Price of Allowance (Demand Increase is 3% a year)		
	\$150	\$300	\$1200
Total Operating Costs (\$)	401,245,441	401,245,441	427,564,704
Total SO ₂ emissions (tons)	60,053	60,053	29,711
Total Cost of Emissions (\$)	9,007,953	18,015,906	35,652,856
Total Unserved Energy (MWh)	0	0	0
Total Investment costs (\$)	0	0	59,896
Total Cost (\$)	410,313,594	419,321,697	463,308,364
Decision	Do-Nothing	Do-Nothing	Switch to Lower Sulfur Coal in t=1

The results are as follows:

1) For the next 10 years GRU does not need to build a new power plant, if the demand growth is less than or equal to 3%.

2) Depending on the price of allowances, the Deerhaven unit (DH2) should operate on lower sulfur coal if the price of an allowance is \$1200 or greater.

The breakpoint for the decision of "Do-Nothing" and switching to lower sulfur is \$920 per allowance.

3) GRU allowances (8300 allowances for 10 years) may cover for some emissions noted above. GRU should buy the extra allowances from the market.

The performance of this plan is shown in table 6.6. Under this compliance

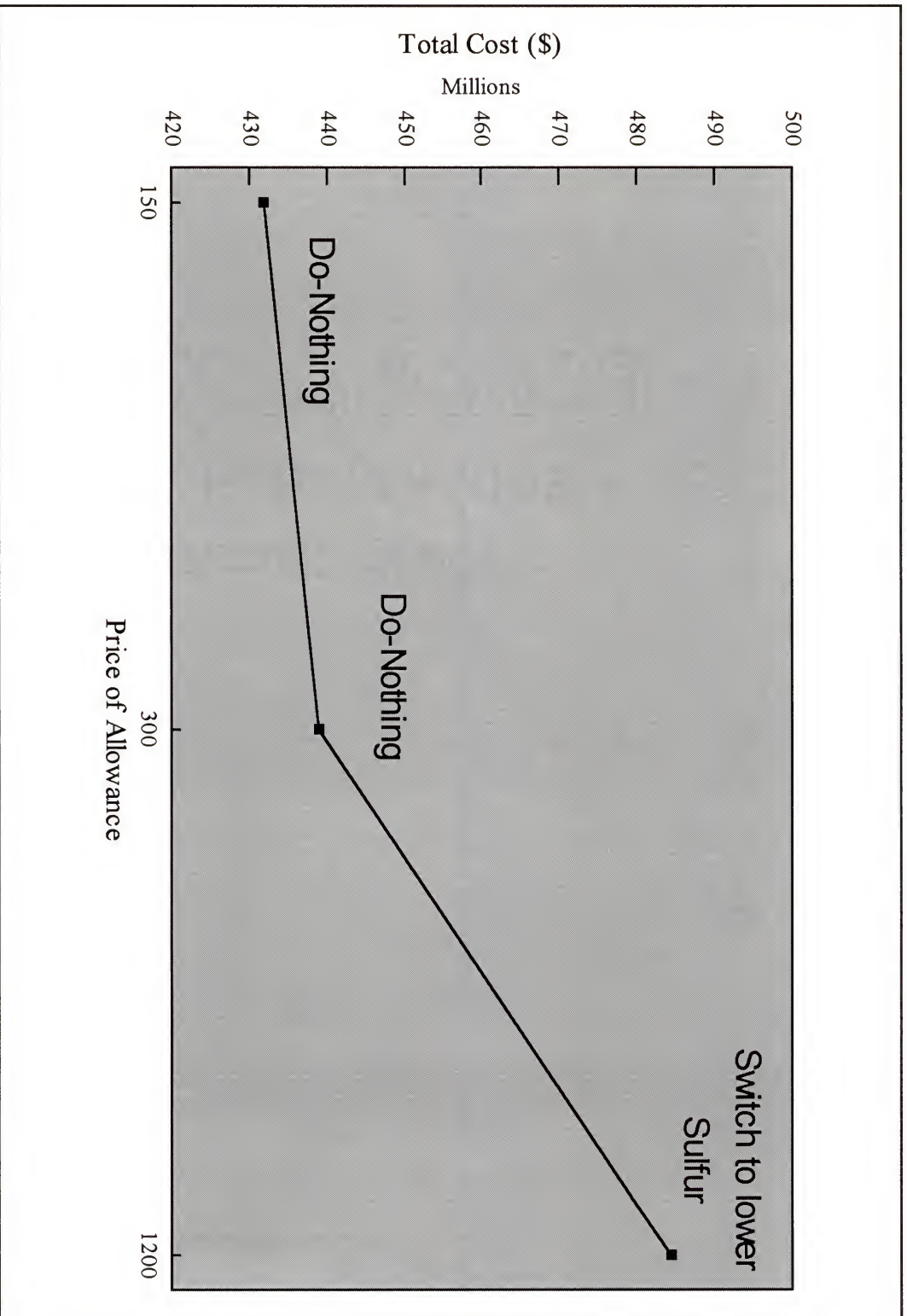


Figure 1 GRU Least Cost Plan Under Different Prices of Allowance

plan, the net cost to the utility is

$$\text{Total cost} \pm \sum P_t * EA_t$$

where the total cost is shown in the table 6.6, P_t price of allowance in period t , EA_t (in tons) is the number of allowances allocated to GRU in period t . The utility's emission allowance will not effect the choice of retrofit options, because it is included in a constant expression in the objective function of the formulation. If the number of allowances exceeds the total emissions, then the utility can sell the excess allowance at the market price; otherwise, the utility needs to buy an additional allowance from the market. In case of overcompliance, that is, the utility's emissions allowance exceeds its total emissions, then the extra emissions allowance is sold in the emissions allowance market for a profit which reduced the utility's compliance cost.

Sensitivity Analysis

The analysis of potential effects on resource selection resulting from variation of forecasted values for significant cost parameters gives insight to the "robustness" of a plan. Numerical results show that the price of emission allowances, demand growth, and the fuel costs have a great impact on the optimal plan. Both allowance prices and fuel prices are assumed fixed in the ECCP model, however, these prices are highly uncertain. By varying the costs of critical parameters, such as fuel prices and emission allowance prices, the effect on which resource options are most economic appears.

In general, there are two approaches used in this aspect of planning; scenario analysis and sensitivity analysis. In scenario analysis key parameters are changed (e.g., fuel prices) and a new optimum plan is determined for each scenario. In each case, this may or may not be the same as the base case optimum plan. The concept of "robustness" is demonstrated for a plan if it is the best or near best for the broadest range of scenarios.

In sensitivity analysis, the base optimum plan is maintained, but its performance under varying parameters is measured. This concept is especially important in determining the effect of binding decisions (e.g. in the reliability constraint if the $EUE(Y_i) = R_i$, the constraint is binding).

Using the combination of both sensitivity and scenario analysis, inferences regarding the risk of a plan are drawn. This is the approach taken by this thesis in the risk assessment portion of this study.

Sensitivity analysis and scenario analysis were conducted for variations in four key parameters:

- 1- Emission Allowance Price Escalation
- 2- Demand Growth Forecast
- 3- Fuel Price Escalation
- 4- Reserve Capacity Requirements

Emission Allowance Price Escalation

The following prices of allowances are considered: \$150/ton, \$300/ton and \$1200/ton. We applied the solution procedures that we discussed in Chapter 4 to find the least cost plan for GRU under different prices of allowances. The results are summarized in tables 6.7, 6.8 and 6.9.

At \$150/ton, the optimal plan for GRU is not to install any emission control option on any power units. There is no incentive for GRU to install any emission reduction option to reduce the SO₂ emission at the allowance price of \$150/ton. The total SO₂ emission as shown in table 6.7 is 48,414 ton for 10 years averaging 4,841 tons per year for a total cost of \$7,262,100.

Table 6.7 GRU Least Cost Plan, P=\$150/ton

Price of Allowance \$150/ton	
Total Operating Costs (\$)	424,769,998
Total SO ₂ emissions (tons)	48,414
Total Cost of Emissions (\$)	7,131,376
Total Unserved Energy (MWh)	0
Total Investment costs (\$)	0
Total Cost (\$)	431,901,374

At \$300/ton, the optimal plan for GRU is also not to install any emission control option on any of its power plants. The price of emissions is not high enough to justify installing a retrofit option. The total cost of SO₂ goes up by 100% (see table 6.8) since the price of the emission allowance is doubled. The total cost to generate electricity to meet the demand for GRU increased by approximately 16.5% .

Table 6.8 GRU Least Cost Plan, P=\$300/ton

Price of Allowance \$300/ton	
Total Operating Costs (\$)	424,769,998
Total SO ₂ emissions (tons)	48,414
Total Cost of Emissions (\$)	14,264,752
Total Unserved Energy (MWh)	0
Total Investment costs (\$)	0
Total Cost (\$)	439,034,750

At \$1200/ton, the optimal plan for GRU along with a "Do-nothing" option are shown in table 6.9. It can be seen that at \$1200/ton, the cost of an allowance is high enough to justify installing a retrofit option to reduce the SO₂ emissions. The total SO₂ emissions decreased by almost 50% (from 48,414 tons to 23,994 tons). The total cost to generate electricity to meet the demand for GRU decreased by approximately 5.7% . Therefore, there is an incentive for GRU to install this emission reduction option at an allowance cost of \$1200/ton. The breakpoint for the decision of "Do-Nothing" and switching to lower sulfur is \$920 per allowance.

Demand Growth Forecast

The variations in load forecast for GRU system were approached from the scenario analysis viewpoint. The intention was to determine the optimal plan for the 3% and 4% growth scenarios. Table 6.10 summarizes the optimal compliance plan

for the next 10 years. Note that the only retrofit option recommended is to use lower sulfur coal on the Deerhaven plant (DH2) when the price of allowances reached \$1200/ton or greater.

Table 6.9 GRU Least Cost Plan, P=\$1200/ton

Price of Allowance	\$1200	\$1200 (Optimal Plan)
Total Operating Costs (\$)	455,630,434	455,630,434
Total SO ₂ emissions (tons)	48,414	23,994
Total Cost of Emissions (\$)	58,096,800	28,792,800
Total Unserved Energy (MWh)	0	0
Total Investment costs (\$)	0	59,896
Total Cost (\$)	513,777,148	484,508,624
If Decision	Do-Nothing	Switch to Lower Sulfur Coal in t=1 (Optimal Plan)

At a demand growth of 4% a year, we reached the same conclusion as above. There is no significant effect on the compliance plan even when the demand increased to 4% (see table 6.11). In summary, the installation of option 3 (lower sulfur coal) is the best choice for GRU regardless of demand forecast variations (3% or 4%).

Table 6.10 Demand Growth Increases by 3% a year

	Price of Allowance (Demand Increase is 3% a year)		
	\$150	\$300	\$1200
Total Operating Costs (\$)	401,245,441	401,245,441	427,564,704
Total SO ₂ emissions (tons)	60,053	60,053	29,711
Total Cost of Emissions (\$)	9,007,953	18,015,906	35,652,856
Total Unserved Energy (MWh)	0	0	0
Total Investment costs (\$)	0	0	59,896
Total Cost (\$)	410,313,594	419,321,697	463,308,364
Decision	Do-Nothing	Do-Nothing	Switch to Lower Sulfur Coal in t=1

Table 6.11 Demand Growth Increases by 4% a year

	Price of Allowance (Demand Increase is 4% a year)		
	\$150	\$300	\$1200
Total Operating Costs (\$)	417,765,194	417,765,194	442,360,375
Total SO ₂ emissions (tons)	60,258	60,258	30,129
Total Cost of Emissions (\$)	9,038,729	18,077,458	36,154,916
Total Unserved Energy (MWh)	0	0	0
Total Investment costs (\$)	0	0	59,896
Total Cost (\$)	426,864,327	435,903,206	478,606,512
Decision	Do-Nothing	Do-Nothing	Switch to Lower Sulfur Coal in t=1

Fuel Price Escalation

Variations in fuel costs were assessed for their effect on the resulting plan.

The fuel price forecast for two primary fuels, natural gas and coal, provided the basis for this sensitivity analysis.

Gas price analysis

Gainesville Regional Utilities has been purchasing natural gas for its generation from the spot market, and will continue this policy in the future. This approach has been beneficial to GRU in the last few years especially because of the depressed prices for natural gas as the nation experienced two successive mild winters. For our analysis we will assume that the price of natural gas will increase by 5% a year from the base year (see table 6.1 for initial prices).

Coal price analysis

Our estimates of low sulfur coal prices indicate that these types of coal will escalate at the general level of inflation throughout the forecast period (Stone and Webster Management Consultants, 1992). The prices of high sulfur coal should escalate at levels slightly lower than inflation and experience values lower than that which will be paid for low sulfur coal. These assumptions are in agreement with projections of demand for low sulfur coal which factor in the impacts of the 1990 Clean Air Amendment.

Chief among the many factors expected to influence the price of coal in future years are the Clean Air Act Amendment, potential Congressional action on global warming, the success of clean coal technologies being tested, and the aging of existing power plants. A resurgence in the demand for nuclear power could also impact the price of coal.

Prices could rise to levels higher than the base rate of inflation primarily due to two factors: shortage of supply and excess of demands. Neither of these conditions is expected in the coal industry within the next twenty years. Shortage of supply would develop if the price of coal fell. However, this could force the price of coal up, thereby inducing new coal mines to open or former mines to reopen. Such price fluctuations are projected to be temporary in nature due to imbalances in supply and demand.

Demand might increase for coal through the study horizon. However, utilities are not currently building new generation, and, in fact, seem primarily involved in

meeting requirements of the Clean Air Act Amendments. Any upward pressures on prices of coal that would occur due to new demand from coal-fired power plants would occur at the beginning of the century (Stone and Webster Management Consultants, 1992).

Absent additional demands which result from clean coal technology use, demand for high sulfur coal is expected to remain flat or to decline. Thus, the price for this type of coal may not even remain at the levels projected in the base case. Medium sulfur coal should experience pockets of local demand at levels which would keep prices at or below levels in the base case. Utilities may choose low sulfur coals in the near term in an effort to avoid installing scrubbers, temporarily increasing the price levels. As the deadline for the second phase of the Clean Air Act Amendment approaches (January 1, 2000), scrubbers might be required on most generating units, offsetting benefits associated with the lower sulfur coals and reducing upward pressures on the price of the lower sulfur coals. For our analysis we will assume that the price of coal will increase by 3% a year from the base year. The results of fuel price escalation run are summarized in table 6.12 (see table 6.1 for initial prices).

Table 6.12 Fuel Price Escalations Run

	Price of Allowance		
	\$300	\$1200	\$1200
Total Operating Costs (\$)	489,938,335	489,938,335	521,852,534
Total SO ₂ emissions (tons)	59,933	60,045	29,665
Total Cost of Emissions (\$)	17,979,965	72,053,626	35,597,802
Total Unserved Energy (MWh)	0	0	0
Total Investment costs (\$)	0	0	59,896
Total Cost (\$)	507,918,300	561,991,961	557,510,232
Decision	Do-Nothing	Do-Nothing	Switch to Lower Sulfur Coal in t=1

Reserve Capacity Requirements

A certain amount of reserve capacity is needed to be able to meet customer required demand. Installed reserves are required to allow for scheduled maintenance, unplanned or forced outages, daily fluctuations in customer demand, and to satisfy operating and spinning reserve requirements. The amount of reserve is primarily a function of the number, size, and reliability of the installed generating units, scheduled maintenance requirements, and the capacity and reliability of transmission interconnections to other utilities.

Many power pools perform detailed reliability analyses to establish the appropriate level of reserves (Stone and Webster Management Consultants, 1992). Typical industry levels of installed reserve requirements range from 15% to 25%.

Based on the reliability and size of the generating units in GRU's system and its external transmission interconnections, GRU believes that a 20% reserve criterion is adequate and reasonable for planning purposes.

In the GRU example, if the demand growth increased by 3% a year, the price of allowances is \$300/ton, the price of natural gas increases by 5%, the price of coal increases by 3% a year and the reserve margin is 20%, the least cost strategy for GRU is not to build any power plant or to install any retrofit control option on the Deerhaven plant (DH2). On the other hand, if the demand growth increased by 4% a year, the price of allowances is \$300/year, the price of natural gas increases by 5%, the price of coal increases by 3% a year and the reserve margin is 20%, GRU should build a new gas turbine power plant in year 9.

Scaled Down Electric Power Research Institute (EPRI) System E

The purpose of this section is to illustrate the solution procedure that was discussed in Chapter 4 with another utility system example that is substantially larger than the GRU system. In the next page, table 6.13 has a list of 36 power generation units for scaled down EPRI system E example along with the source fuel of each unit, the capacity in MW of each unit, the outage rate, the fuel prices and the emission rate in lb/MWh. In table 6.14 we list the new characteristics of the retrofitted units (3,4,5,6,7) with SO₂ control options.

Table 6.13 System Data

Unit	Source	Capacity (MW)	Forced Outage Rate	Operating Cost (\$/MWh)	SO ₂ Emission Rate ((lb/MWh)
1	Nuclear	1200	0.132	10	00
2	Nuclear	1200	0.132	10	0
3	Coal	600	0.153	15	60
4	Coal	600	0.153	18	60
5	Oil	400	0.119	50	30
6	Oil	400	0.119	55	30
7	Oil	600	0.119	50	25
8	Gas	200	0.082	55	0
9	Gas	200	0.082	55	0
10	Gas	200	0.082	55	0
11	Gas	200	0.082	50	0
12	Gas	400	0.171	60	0
13	Gas	400	0.171	60	0
14	Gas	200	0.082	55	0
15	Gas	200	0.082	65	0
16	Gas	200	0.082	60	0
17	Gas	50	0.12	70	0
18	Gas	50	0.12	70	0
19	Gas	50	0.12	70	0
20	Gas	50	0.12	70	0
21	Gas	50	0.12	70	0
22	Gas	50	0.12	70	0
23	Gas	50	0.12	70	0
24	Gas	50	0.12	70	0

Unit	Source	Capacity (MW)	Forced Outage Rate	Operating Cost (\$/MWh)	SO ₂ Emission Rate ((lb/MWh)
25	Gas	50	0.12	70	0
26	Gas	50	0.12	70	0
27	Gas	50	0.12	70	0
28	Gas	50	0.12	70	0
29	Gas	50	0.12	70	0
30	Gas	50	0.12	70	0
31	Gas	50	0.12	70	0
32	Gas	50	0.12	70	0
33	Gas	50	0.12	70	0
34	Gas	50	0.12	70	0
35	Gas	50	0.12	70	0
36	Gas	50	0.12	70	0

Table 6.14 New System Characteristics of Power Unit

Unit	Retrofit Type	Operating Cost (\$/MWh)	SO ₂ Emission Rate (lb/MWh)	Fixed Cost	New Capacit y (MW)
3	FGD	19	3	2,825,571	570
3	LS	17	30	80,000	594
3	DLI1	21	45	2,030,800	575
4	LS	20	30	70,000	570
5	LIMB	53	15	500,000	360
6	LIMB	53	15	480,000	360
7	LIMB	53	12	450,000	360

In this section, we apply the solution procedure that we described in Chapter 4 to find the least cost expansion and emission plan. In this example, we will assume the price of emission allowances is \$300/ton. Considering a retrofit on plants 3-7 and three different expansion options, the least cost was found by applying the partitioning solution procedure that was developed in Chapter 4 and summarized in appendix C. The findings are summarized in Table 6.15.

Table 6.15 Least Cost Plan

Plant Number	Retrofit Option	Expansion Option
3	FGD in year 1	do-nothing
4	Low Sulfur Coal in year 1	
5	do-nothing	
6	do-nothing	
7	do-nothing	

Therefore, the least cost plan under a price of allowances of \$300/ton is the following:

- 1- Install retrofit option FGD on unit 3 at period 1.
- 2- Install retrofit option LS on unit 4 at period 1.

The performance of this plan is shown in table 6.16.

Table 6.16 Performance of the Least Cost Plan

Total SO ₂ emissions (tons)	593,413
Total Cost of Emissions (\$)	178,024,030
Operating Costs (\$)	2,923,128,271
Investment Costs (\$)	2,895,571
Expansion Costs (\$)	0
Total Costs (\$)	3,104,047,872

It is found that the partitioning solution procedure terminates in the 9th iteration. Nine relaxed master problems and nine operating subproblems are solved. The relaxed master problems are solved using the software package LINDO.

The performance of the partitioning solution procedure is demonstrated in Figure 6.1. Figure 6.1 shows the upper bound (UB) and lower bound (LB) for each iteration.

It can be seen from Figure 6.1 that the lower bound increases at each iteration towards the optimal value. This is because at each iteration, one additional constraint is added into the relaxed master problem. Figure 6.1 also shows that the upper bound converges to the optimal value.

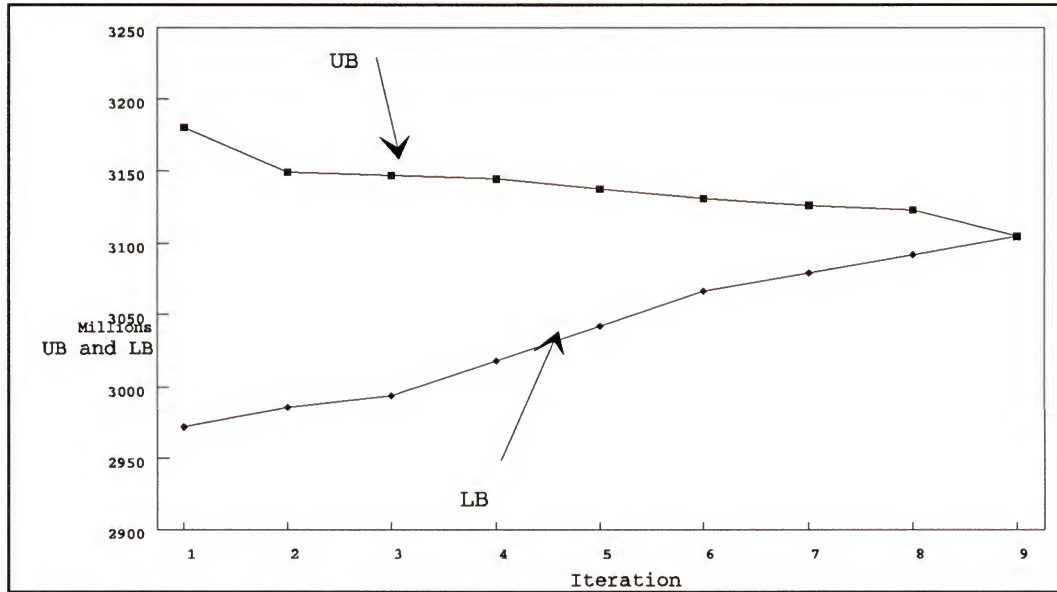


Figure 2 Iteration VS UB and LB

Third Example (Huang and Hobbs Example)

This section demonstrates the solution procedures to a synthetic utility system, called Scaled Down EPRI System D. This system has been used before to study the one-period emission control system by Huang and Hobbs, (1994). The description of the system is given in the next section. The solution of the system with the new formulation is found and a comparison between the presented solution and Huang and Hobbs solution will be discussed in the last section.

Description of the System

Table 6.17 is a list of unit information of the system. In that table we list the unit source, fuel type, number of units, capacity of each unit (MW) and the forced

Table 6.17 Unit Information

Sources	Fuel Types	Number of Units	Unit Capacity (MW)	Forced Outage Rate	Operating Cost \$/MWh	SO ₂ Emission Rate lb/MWh
1	Nuclear	2	1200	0.15	10	0
2	Coal	1	800	0.24	15	63.33
3	Coal	2	600	0.21	18	50
4	Coal	2	400	0.13	20	33.33
5	Oil	8	400	0.13	50	25.2
6	Oil	8	200	0.074	55	20
7	Gas	7	200	0.074	60	0
8	Gas	29	50	0.24	70	0

The system load has a mean of 6267.4 MW and a standard deviation of 745.4 MW. The period over which the units are dispatched is 8760 hours.

The possibility of fuel switching and retrofit of emissions control equipment is considered for units 3,4,5, and 7. The effect of the retrofit on the generation units can be seen in table 6.18.

We considered three different options for building a new power plant; this was not considered in Huang and Hobbs model. We will consider each option in every period of the time horizon of our analysis. Table 6.19 shows the plant type that is under consideration to be built, the size of the proposed plant, the forced outage rate, the operating cost in \$/MWh, the emission rate (lb/MWh), and the cost of building the proposed unit.

Table 6.18 New Unit Characteristics of Affected Units

Unit	Retrofit Type	New Unit Capacity (MW)	Operating Cost \$/MWh	SO ₂ Emission Rate (lb/MWh)	Fixed Cost
3	FGD	768	20.5	3.33	2,825,571
3	LS	792	19.25	31.67	79,781
3	DLI1	768	22.5	9.98	2,027,763
3	DLI2	768	20.5	19.95	1,628,858
3	DLI3	768	19.5	19.95	565,114
4	LIMB	576	24	16.67	485,333
5	LIMB	576	24	16.67	485,333
6	LIMB	384	24	16.67	322,447
7	LIMB	384	24	16.67	322,447

Table 6.19 Expansion Data

Plant type	Size (MW)	Forced Outage (%)	Operating Cost (\$/MWh)	SO ₂ Emission Rate (lb/MWh)	Cost (\$)
Gas Turbine	500	15	35	0	14,000,000
Gas Turbine	1000	15	30	0	28,000,000
Coal	1000	10	15	13	76,000,000

The Least Cost Plan

In this section, we will compare the optimal solution obtained using the new approach with those of Huang and Hobbs and demonstrate the procedure for finding the least costs plan. Table 6.20 shows the results and the least cost plan for the utility system found by the Huang and Hobbs method and by our method.

It is found that the partitioning solution procedure using our method terminates in the 15th iteration. Fifteen relaxed master problems and fifteen operating subproblems are solved. The relaxed master problems were solved using the software package LINDO. Huang and Hobbs solved the problem over one period (one month). In their model, the reliability constraints were not considered, nor was an expansion to build a new power plant considered. Instead the assumption for the expected unserved energy was penalized by \$100 per MWh not served. This penalty represents the value customers attached to power lost during outages. The same retrofit option was found by both methods, DLI3 on unit 3. The utility monthly average cost

determined by the Huang and Hobbs model is \$149,131,709. On the other hand, the utility monthly average cost using our model is \$200,207,643. In our method, the cost includes not only the operating and emission costs but also the cost of building two additional power plants to meet the demand and the reserve margin of 20% in the next five years. Our formulation includes new components beyond that in the Huang and Hobbs model in order to achieve the required level of realism. These components are summarized in the second section of Chapter 7.

Table 6.20 Least Cost Plan Comparison

The optimal solution based on an increase in demand of 4% a year, reserve margin of 20% a year, minimum run constrain on all existing units of 20% of the capacity and the horizon of the study is based on 5 years.		Huang and Hobbs The problem was formulated for only one period (728 hours), the price of allowance is \$500/ton, no minimum run constraints were considered, demand was assumed to be constant and no reserve margin considered.
Operating Costs (\$)	11,787,660,877/5years	128,882,994/month
Total SO ₂ emissions (tons)	2,684,653/years	36,725
Total Cost of Emissions (\$)	134,232,643/5years	18,362,500/month
Total Unserved Energy (MWh)	0	1,320,886
Total Investment costs (\$)	90,565,114	565,114
Total Cost (\$)	12,012,458,634	149,131,494/month
Decision	Build 1000 MW of coal power plant in year 2, build 500 MW of gas turbine power plant in year 5 and install DLI1 in year 1 on power plant 3	Install retrofit option DLI3 on power plant 3

CHAPTER 7

SUMMARY AND CONCLUSION

This theses presents a pragmatic approach to solving realistic Emission Capacity Controls Problem (ECCP) using modern interactive computing technology such as a spreadsheet. This goal gave rise to a system engineering approach where the effort was balanced among problem formulation, solution techniques and solution implementation. In this sense the contribution of this dissertation is in bringing together such key aspects rather than a narrow but deeper study in any one given aspect.

The purpose of this chapter is to summarize the contribution of this dissertation and discuss the further potential research in the area of scheduling and optimization for the electric utilities.

Summary and Contribution

The major contribution of this thesis is the presentation and solution of the Emission Capacity Controls Problem (ECCP). Our formulation includes new components beyond that in the literature in order to achieve the required level of realism. These components are briefly summarized below.

1- the ECCP problem was formulated for a multi-year.

Since the demands are growing, fuel prices changing, allowance prices changing, therefore a one period solution may not be optimal for a changing system. A multi-year nonlinear optimization model was constructed. The objective function is the sum of the total cost of investment, operation, and SO₂ emission.

2- reliability constraints were added to the formulation.

A primary mandate of an electric utility is to provide electricity in sufficient supply and in sufficient quality to satisfy its customers' needs and expectations. Sufficient supply and quality means providing it at a reasonable level of reliability and as economically as possible. However, it is difficult to define that "reasonable" level of reliability and it has often been established somewhat arbitrarily. Typical utility of installed requirements range from 15% to 25%.

3- expansion of a utility system.

An important characteristic of the demand for electricity is that it is highly time-dependent, varying considerably from year to year. Thus, though enough generating capacity must be built to meet the demand. Therefore, minimizing the cost of meeting the demand involves trading-off the capital costs of plants against their operating costs.

As a result of demand growing, utilities must plan on building a new power unit so as to meet the growing demand.

4- minimum-run constraints.

Many utilities subject some of their units to run at least some percentage of the time during a period. The spreadsheet is easily programmed to take that into consideration. The minimum run constraint on specific unit will only affect the subproblem and it has no effect of the relaxed problem.

Regarding the solution procedure and the implementation, a genuine effort was made to assure that useful results can be obtained in a reasonable time on a small computer such as a portable PC. The spreadsheet portion which solves the subproblem bring many innovation elements to solution implementation. Spreadsheets are inherently interactive besides being programmable in macro language, the user may pause the spreadsheet, view or plot the intermediate results and make modifications to variables. This later point alone is a power feature in playing "what if games" as it is commonly referred to in industry. The key elements of the solution procedure and implementation are briefly summarized below.

- 1- Find the expected energy produced using the production simulation method within the spreadsheet.
- 2- Solve the subproblem by a spreadsheet.
- 3- Use the interactive spreadsheet "what if games"
- 4- Find derivatives of production cost function with respect to all constraints (reliability and capacities) in a spreadsheet.

The ECCP model can be used to aid the utilities in making decisions regarding the expansion of their system as well as planning the SO₂ compliance. The success of the model may potentially save millions of dollars in the SO₂ compliance and expansion costs. Furthermore, a method is developed to evaluate the derivatives of the production costing function. The derivatives are used in the partitioning solution procedure that was developed in Chapter 4. Finally, we demonstrated the successful use of the partitioning solution procedures in solving practical problems.

In Chapter 3 we developed a formulation of the Emission Capacity Controls Problem (ECCP). The optimization of power system operation over periods of time requires simulation model based on probability analysis which combines reasonable accuracy with high computation speed. The probabilistic production costing method is used to calculate the generation unit output and the cost of the output. This method is a standard means used by utilities.

The decision variables are the choices of retrofit and expansion options for the generations units. Four sets of constraints are considered. The first set of constraints limits the operating level of each power unit to be less than or equal to its capacity. The second set of constraints states that the unserved amount of energy must be less than or equal to the reliability level. The third set of constraints is the "must run constraints" which states that a particular unit must run at least some specified percentage of time. The last set of constraints states that only one retrofit option can be chosen for each unit.

In Chapter 4 we derived solution procedures to solve the Emission Capacity Controls Problem (ECCP). In that chapter we showed a feature peculiar to the problem. That is, when all the X (the zero-one decision variables) values are given in advance, the remaining problem is the operating problem for the generation system. We showed how this allows us to apply the partitioning approach to find the solution for the ECCP problem.

The underlying idea of the partitioning approach used in this model is to separate the ECCP into two parts: a master problem that generates trial solutions for the optimal emission capacity plan; and a subproblem that determines the optimal operating scheme for each trial-generating system.

Future Research

In our model that was presented in Chapters 3 and 4, we used some assumptions in order not to obscure its basic structure. These assumptions are:

- 1- Hydroelectric or other nonthermal plants are not considered. We only considered thermal power plants.
- 2- Only plants of certain sizes and certain types are considered for expansion options.
- 3- No retrofit of an SO_2 option is considered for the new power plants built.
- 4- The demand uncertainty is considered by having different scenarios of forecasted demand, and the problem is re-solved for each scenario.

Many of these features could be included in the model without disrupting its

structure, and the iterative solution procedures can still be applied. In the next section we will discuss how some of these assumptions and limitations could be handled.

Hydroelectric Power Plants

One of the major differences between thermal power plants and hydroelectric power plants is that the total power generated by a hydroelectric power plant depends on the water stored behind the dam. Jacoby (1967) developed a special methodology for placing hydroelectric power plants in merit order. Since the operating cost of the hydroelectric power plants, in general, is negligible, it would be desirable to place them at the bottom of the loading order. However, by doing so, we will have the problem of expecting energy that may not be available at a certain time because the energy generated depends on the amount of water stored behind the dam. Jacoby's rule states that a hydroelectric power plant of a certain capacity and with given stored energy should be placed in the merit order so that the total energy generated is exactly equal to the stored energy. The merit order of the operation is still the optimal strategy, and the solution for the subproblem proceeds as before.

The partitioning principle can also handle the situation when the hydroelectric energy stored in each period is a decision variable. In this case, the amounts of energy available are included as decision variables in the master problem and passed as data to the subproblem along with plant capacities. More constraints are included in the master problem. The amount of water stored behind the dam at the end of period t plus the amount of energy available should be equal to the water inflow

during period t and the amount of water stored behind the dam in period $t-1$.

Fixed-Size Plants and Their Costs to Build

The ECCP model discussed previously has assumed that the cost of building a new power plant is proportional to its capacity. However, real generating plants exhibit economics of scale: the cost per kilowatt decreases as the plant size increases. Furthermore, plants are usually built in fixed unit sizes because components such as generators are manufactured that way. Cost functions which exhibit economies scale are nonlinear. These cost functions can be approximated piecewise linearly.

No Retrofit of an SO₂ Option for the New Plant

One of the limitations of the ECCP model is that for the new power plant we do not consider a retrofit option to reduce the SO₂ emissions. This limitation can be easily handled by re-solving the ECCP problem with the new power plant already considered as part of the generation system. Also, we can limit our selection to only SO₂ emission free power plants. This limitation and its implementation are topics for future research.

Demand Uncertainty

The ECCP model proposed in Chapters 3 and 4 considers the uncertain aspects of supplying electricity, i.e. the random outage of power plants. However, the demand for electricity is also uncertain, and the impact of this uncertainty is at least

as great as that of supply uncertainty on planning. There are two types of uncertainty in demand, long-term and short-term. The long-term uncertainty is primarily concerned with the gross parameters of demand, such as yearly peak, yearly energy, and load factor several years ahead. The short-term uncertainty is primarily concerned with the smaller variations which will occur hourly or daily over the next year or so. Both types of uncertainty are really different aspects of the same thing, but different methods can be used to deal with them. For planning purposes, long-term uncertainty can be dealt with by considering different scenarios. Usually, a small number of scenarios will be sufficient to represent the bound of the expected variation in long-term forecasts. For each scenario, a demand forecast is developed, and the ECCP problem is re-solved. Then, based on the likelihood of each scenario occurring, one or a combination of the plans is chosen.

In order to deal with short-term uncertainty, it is necessary to find a way to represent the short-term random characteristics of the demand profile. To a certain extent, the load duration curve already represents some of these characteristics. Although the load duration curve is often used as one, it is not exactly a probability distribution. Thus, it would be useful to be able to include the random characteristics of demand within the load duration curve. Future research may need to address such limitation.

APPENDIX A SPREADSHEET PROGRAM

A:A1: {Page 1/1} U 'UNITS
 A:B1: {Page} U [W12] "\$/MWH
 A:C1: {Page} U [W12] "STD. CAPACITY
 A:D1: {Page} U [W12] "EXPECTED MW
 A:E1: {Page} U [W12] "P
 A:F1: {Page} U [W12] "q
 A:G1: {Page} [W10] $\wedge_{g1i} = M1i$
 A:H1: {Page} "M2i
 A:I1: {Page} "g2i
 A:J1: {Page} "M3i
 A:K1: {Page} "g3i
 A:L1: {Page} "M4i
 A:M1: {Page} (F0) "g4i
 A:A2: {Page} 'CR3
 A:B2: U [W12] 5
 A:C2: U [W12] 11
 A:D2: (F0) U [W12] @ROUND(Q8,0)
 A:E2: U [W12] 1-F2
 A:F2: U [W12] 0.276
 A:G2: [W10] +F2*C2
 A:H2: +G2*C2
 A:I2: +H2-G2²
 A:J2: +C2*H2
 A:K2: +J2-3*G2*H2+2*G2³
 A:L2: +C2*J2
 A:M2: (F0) +L2+6*H2*G2²-4*J2*G2-3*G2⁴-3*I2²
 A:A3: {Page} U 'DH2
 A:B3: U [W12] 21
 A:C3: U [W12] 235
 A:D3: (F0) U [W12] @ROUND(Q9,0)
 A:E3: U [W12] 1-F3
 A:F3: U [W12] 0.13
 A:G3: [W10] +F3*C3
 A:H3: +G3*C3
 A:I3: +H3-G3²

A:J3: $+C3*H3$
 A:K3: $+J3-3*G3*H3+2*G3^3$
 A:L3: $+C3*J3$
 A:M3: $(F0) +L3+6*H3*G3^2-4*J3*G3-3*G3^4-3*I3^2$
 A:A4: {Page} U 'JRK8
 A:B4: U [W12] 38.3
 A:C4: U [W12] 44
 A:D4: (F0) U [W12] @ROUND(Q10,0)
 A:E4: U [W12] 1-F4
 A:F4: U [W12] 0.15
 A:G4: [W10] $+F4*C4$
 A:H4: $+G4*C4$
 A:I4: $+H4-G4^2$
 A:J4: $+C4*H4$
 A:K4: $+J4-3*G4*H4+2*G4^3$
 A:L4: $+C4*J4$
 A:M4: $(F0) +L4+6*H4*G4^2-4*J4*G4-3*G4^4-3*I4^2$
 A:A5: {Page} U 'DH1
 A:B5: U [W12] 38.5
 A:C5: U [W12] 81
 A:D5: (F0) U [W12] @ROUND(Q11,0)
 A:E5: U [W12] 1-F5
 A:F5: U [W12] 0.129
 A:G5: [W10] $+F5*C5$
 A:H5: $+G5*C5$
 A:I5: $+H5-G5^2$
 A:J5: $+C5*H5$
 A:K5: $+J5-3*G5*H5+2*G5^3$
 A:L5: $+C5*J5$
 A:M5: $(F0) +L5+6*H5*G5^2-4*J5*G5-3*G5^4-3*I5^2$
 A:A7: {Page} "ELK1
 A:B7: [W12] "ELK2
 A:C7: [W12] "ELK3
 A:D7: [W12] "ELK4
 A:E7: [W12] "U
 A:F7: [W12] "SIG^2
 A:G7: [W10] "G1
 A:H7: "G2
 A:I7: "Z
 A:J7: 'SUM CAP
 A:K7: "N(Z)
 A:L7: ^INT(Z)
 A:M7: "N2
 A:N7: "N3

A:O7: "N5
 A:P7: [W14] "ELC
 A:A8: {Page} (F0) + \$A\$592 + G2
 A:B8: (F0) [W12] + A593 + I2
 A:C8: (F0) [W12] + A594 + K2
 A:D8: (F0) [W12] + A595 + M2
 A:E8: (F0) [W12] + A8
 A:F8: (F0) [W12] + B8
 A:G8: (F3) [W10] + C8/@SQRT(F8)^3
 A:H8: (F3) + D8/F8^2
 A:I8: (F2) (J8-E8)/@SQRT(F8)
 A:J8: + C2
 A:K8: (F4) U 0.39894228*@EXP(-0.5*(I8^2))
 A:L8: (F4) U @VLOOKUP(@ABS(I8), \$CN\$1..\$CO\$100, 1)
 A:M8: (F4) (I8^2-1)*K8
 A:N8: (F4) (-1*I8^3+3*I8)*K8
 A:O8: (F4) (-1*I8^5+10*I8^3-15*I8)*K8
 A:P8: (F4) [W14] 1-\$L8+(\$G8*M8)/6-(\$H8*N8)/24-(10*\$G8^2*O8)/720
 A:Q8: +E2*C2*((P8+1)/2)
 A:A9: {Page} (F0) + A8 + G3
 A:B9: (F0) [W12] + B8 + I3
 A:C9: (F0) [W12] + C8 + K3
 A:D9: (F0) [W12] + D8 + M3
 A:E9: (F0) [W12] + A9
 A:F9: (F0) [W12] + B9
 A:G9: (F3) [W10] + C9/@SQRT(F9)^3
 A:H9: (F3) + D9/F9^2
 A:I9: (F2) (J9-E9)/@SQRT(F9)
 A:J9: + J8 + C3
 A:K9: (F4) U 0.39894228*@EXP(-0.5*(I9^2))
 A:L9: (F4) U @VLOOKUP(@ABS(I9), \$CN\$1..\$CO\$100, 1)
 A:M9: (F4) (I9^2-1)*K9
 A:N9: (F4) (-1*I9^3+3*I9)*K9
 A:O9: (F4) (-1*I9^5+10*I9^3-15*I9)*K9
 A:P9: (F4) [W14] 1-\$L9+(\$G9*M9)/6-(\$H9*N9)/24-(10*\$G9^2*O9)/720
 A:Q9: +E3*C3*((P9+P8)/2)
 A:A10: {Page} (F0) + A9 + G4
 A:B10: (F0) [W12] + B9 + I4
 A:C10: (F0) [W12] + C9 + K4
 A:D10: (F0) [W12] + D9 + M4
 A:E10: (F0) [W12] + A10
 A:F10: (F0) [W12] + B10
 A:G10: (F3) [W10] + C10/@SQRT(F10)^3
 A:H10: (F3) + D10/F10^2

A:I10: (F2) $(J10-E10)/@SQRT(F10)$
 A:J10: $+J9+C4$
 A:K10: (F4) $U\ 0.39894228* @EXP(-0.5*(I10^2))$
 A:L10: (F4) $U\ @VLOOKUP(@ABS(I10),\$CN\$1..\$CO\$100,1)$
 A:M10: (F4) $(I10^2-1)*K10$
 A:N10: (F4) $(-1*I10^3+3*I10)*K10$
 A:O10: (F4) $(-1*I10^5+10*I10^3-15*I10)*K10$
 A:P10: (F4) $[W14]\ 1-\$L10+(\$G10*M10)/6-(\$H10*N10)/24-(10*\$G10^2*O10)/720$
 A:Q10: $+E4*C4*((P10+P9)/2)$
 A:A11: {Page} (F0) $+A10+G5$
 A:B11: (F0) $[W12]\ +B10+I5$
 A:C11: (F0) $[W12]\ +C10+K5$
 A:D11: (F0) $[W12]\ +D10+M5$
 A:E11: (F0) $[W12]\ +A11$
 A:F11: (F0) $[W12]\ +B11$
 A:G11: (F3) $[W10]\ +C11/@SQRT(F11)^3$
 A:H11: (F3) $+D11/F11^2$
 A:I11: (F2) $(J11-E11)/@SQRT(F11)$
 A:J11: $+J10+C5$
 A:K11: (F4) $U\ 0.39894228* @EXP(-0.5*(I11^2))$
 A:L11: (F4) $U\ @VLOOKUP(@ABS(I11),\$CN\$1..\$CO\$100,1)$
 A:M11: (F4) $(I11^2-1)*K11$
 A:N11: (F4) $(-1*I11^3+3*I11)*K11$
 A:O11: (F4) $(-1*I11^5+10*I11^3-15*I11)*K11$
 A:P11: (F4) $[W14]\ 1-\$L11+(\$G11*M11)/6-(\$H11*N11)/24-(10*\$G11^2*O11)/720$
 A:Q11: $+E5*C5*((P11+P10)/2)$

APPENDIX B
ILLUSTRATION OF THE SOLUTION PROCEDURES
APPLIED TO GRU SYSTEM

Iteration 1

Subproblem 1

minimize $\sum \sum \text{FEG}(Y) + \sum \sum \text{PSEG}(Y)$

subject to

$$\text{EUE}(Y) \leq R \quad \text{for all } t$$

$$Y \leq \text{Cap } X \quad \text{for all } ij \text{ and } t$$

$$Y \geq 0 \quad \text{for all } ij \text{ and } t$$

start with the initial feasible solutions. All $X_{ijt} = 0$

	Operating Cost & Emission Cost (\$)
1	36,479,816
2	38,490,084
3	40,562,878
4	42,665,074
5	44,920,332
6	47,189,602
7	49,517,176
8	51,165,767
9	54,127,436
10	56,291,656
Total	461409821.3
Investment Cost	0
Total Costs	461,409,821

Relaxed Master 1

Min Z

S.T

$$\sum_i \sum_t (C_{ijt} - W_{ijt} \text{Cap}_{ijt}) X_{ijt} + \sum_i \sum_t W_{ijt} \text{Cap}_{ijt} X_{ijt} + \sum_t \sum_{ij} F_{ijt} \text{EG}(Y_{ijt}) + \sum_t \sum_{ij} P_t S_{ijt} \text{EG}(Y_{ijt}) \leq$$

where

$$W_{ijt} = -[F_{ijt} + P_t S_{ijt}] \partial \text{EG}(Y)/\partial Y_{ijt} - U_t \partial \text{EUE}(Y)/\partial Y_{ijt}$$

X_{ijt} decision variable = $\begin{cases} 1 & \text{if unit } i \text{ is retrofitted with option } j \text{ at time } t \text{ is selected} \\ 0 & \text{else} \end{cases}$

C_{ijt} the investment cost when unit i is retrofitted with option j at time t .

- Cap_{ijt} Capacity of unit i with a retrofit j in period t
 P_t The price of each allowance assumed = \$1200 for all years
 S_{ijt} The emission rate of unit i with a retrofit option of j in period j

In order to find W_{ijt} we have first to calculate the derivatives of EG and EUE with respect to Y

	W30	cost-wcap	W*Cap	W32	cost-wcap	W*Cap	W34	cost-wcap	W*Cap	W36	cost-wcap	W*Cap
1	-329841	77512593	-77512593	-334887	77859080	77358954	-355853	81020937	-80422747	-335071	75892699	-75725990
2	-311510	73204756	-73204756	-316586	73631447	73131321	-337675	76912773	-76314583	-316770	71756819	-71590110
3	-287206	67493504	-67493504	-292301	68021715	67521589	-313468	71441966	-70843776	-292487	66268662	-66101953
4	-235375	55313011	-55313011	-238722	55644908	55144782	-252629	57692406	-57094216	-238844	54145391	-53978682
5	-374560	88021530	-88021530	-379656	88200659	87700533	-400829	91185486	-90587296	-379841	86010844	-85844135
6	-369052	86727127	-86727127	-374086	86913931	86413805	-395000	89868248	-89270058	-374269	84751457	-84584748
7	-332895	78230296	-78230296	-337856	78544867	78044741	-358467	81611808	-81013618	-338036	76562942	-76396233
8	-346704	81475464	-81475464	-351583	81715793	81215667	-371852	84636849	-84038659	-351760	79664556	-79497847
9	-187542	44072468	-44072468	-188787	44109883	43609757	-193957	44432423	-43834233	-188832	42842759	-42676050
10	-222394	52262575	-52262575	-225442	52577196	52077070	-238105	54409832	-53811642	-225553	51141618	-50974909
	W31	cost-wcap	W*Cap	W33	cost-wcap	W*Cap	W35	cost-wcap	W*Cap	W37	cost-wcap	W*Cap
1	-333006	76092968	-75259425	-325712	75950774	75890878	-344246	78280126	-77799613	-344063	77949039	-77758141
2	-314694	71954334	-71120791	-307356	71673923	71614027	-326000	74156483	-73675970	-325815	73825151	-73634253
3	-290402	66464454	-65630911	-283038	66007723	65947827	-301750	68675986	-68195473	-301565	68344501	-68153603
4	-237474	54502735	-53669192	-232636	54264006	54204110	-244930	55834707	-55354194	-244808	55517582	-55326684
5	-377756	86206503	-85372960	-370390	86360768	86300872	-389107	88418759	-87938246	-388922	88087262	-87896364
6	-372209	84952862	-84119319	-364933	85089232	85029336	-383422	87133830	-86653317	-383239	86802844	-86611946
7	-336007	76771095	-75937552	-328836	76678627	76618731	-347057	78915327	-78434814	-346876	78584940	-78394042
8	-349764	79880316	-79046773	-342712	79911861	79851965	-360631	81983134	-81502621	-360454	81653423	-81462525
9	-188323	43394541	-42560998	-186524	43520050	43460154	-191095	43667902	-43187389	-191049	43368060	-43177162

The relaxed problem is a linear programming problem. We used LINDO software, which is a linear programming optimization software, to solve the relaxed problem.

The output of LINDO is
 X_{321} and $X_{ijt} = 0$ for all other ijt .

$$LB = \sum \sum (C_{ijt} - W_{ijt} \text{Cap}_{ijt}) X_{ijt} + \sum \sum W_{ijt} \text{Cap}_{ijt} X_{ijt} + \sum \sum F_{ijt} \text{EG}(Y_{ijt}) + \sum \sum P_t S_{ijt} \text{EG}(Y_{ijt})$$

$$LB = \$463,308,364$$

Is UB = LB No

New Variables

X_{321}

Iteration 2

Subproblem

minimize $\sum \sum \text{FEG}(Y) + \sum \sum \text{PSEG}(Y)$

subject to

$$\begin{array}{ll} \text{EUE}(Y) \leq R & \text{for all } t \\ Y \leq \text{Cap } X & \text{for all } ij \text{ and } t \\ Y \geq 0 & \text{for all } ij \text{ and } t \end{array}$$

Year	Operating Cost & Emission Cost (\$)
1	35,613,213
2	37,640,132
3	39,730,151
4	41,896,160
5	44,121,881
6	46,405,712
7	48,748,234
8	51,165,767
9	53,250,744
10	55,402,760
Total	463,248,468
Investment Cost	59,896
Total Costs (UB)	463,308,364

LB= UB

STOP

The optimal solution is X_{321}

The operating Cost = \$427,564,704

The Emission Cost = \$29,711

Investment Cost = \$59,896

Total Cost = \$463,308

APPENDIX C ILLUSTRATION OF THE SOLUTION PROCEDURES APPLIED TO EPRI SYSTEM E PROBLEM

The purpose of this appendix is to illustrate the solution procedures that was discussed in Chapter 4. We applied the solution procedures to the problem that is described in Chapter 6. Table C.1 is a list of 36 power units for an EPRI example along with the source fuel of each unit, the capacity in MW of each unit, the outage rate, the fuel prices and the emission rate in lb/MWh. In table C.2 we list the new characteristics of the retrofitted unit with a control option.

Table C.1 System Characteristics

Unit	Source	Capacity (MW)	Forced Outage Rate	Operating Cost (\$/MWh)	SO ₂ Emission Rate (lb/MWh)
1	Nuclear	1200	0.132	10	00
2	Nuclear	1200	0.132	10	0
3	Coal	600	0.153	10	60
4	Coal	600	0.153	10	60
5	Oil	400	0.119	50	30
6	Oil	400	0.119	50	30
7	Oil	400	0.119	50	25
9	Gas	200	0.082	55	0
9	Gas	200	0.082	55	0
10	Gas	200	0.082	55	0
11	Gas	200	0.082	55	0

Table C1--continued

Unit	Source	Capacity (MW)	Forced Outage Rate	Operating Cost (\$/MWh)	SO ₂ Emission Rate ((lb/MWh)
12	Gas	800	0.171	60	0
13	Gas	800	0.171	60	0
14	Gas	200	0.082	60	0
15	Gas	200	0.082	65	0
16	Gas	200	0.082	60	0
17	Gas	50	0.12	70	0
18	Gas	50	0.12	70	0
19	Gas	50	0.12	70	0
20	Gas	50	0.12	70	0
24	Gas	50	0.12	70	0
28	Gas	50	0.12	70	0
29	Gas	50	0.12	70	0
24	Gas	50	0.12	70	0
29	Gas	50	0.12	70	0
26	Gas	50	0.12	70	0
27	Gas	50	0.12	70	0
28	Gas	50	0.12	70	0
29	Gas	50	0.12	70	0
30	Gas	50	0.12	70	0
31	Gas	50	0.12	70	0
32	Gas	50	0.12	70	0
33	Gas	50	0.12	70	0
34	Gas	50	0.12	70	0
35	Gas	50	0.12	70	0
36	Gas	50	0.12	70	0

Table C.2 New System Characteristics

Unit	Name	F (\$/MWh)	S (lb/MWh)	Fixed Cost	New Capacity (MW)
4	FGD	19	3	2,825,571	570
4	LS	17	30	80,000	594
3	DLI1	21	45	2,030,800	575
4	LS	20	30	70,000	570
5	LIMB	53	19	500,000	360
4	LIMB	53	15	480,000	360
7	LIMB	53	12	450,000	360

Iteration 1

Subproblem 1

minimize $\sum \sum \text{FEG}(Y) + \sum \sum \text{PSEG}(Y)$

subject to

$$\begin{array}{ll}
 \text{EUE}(Y) \leq R & \text{for all } t \\
 Y \leq \text{Cap } X & \text{for all } ij \text{ and } t \\
 Y \geq 0 & \text{for all } ij \text{ and } t
 \end{array}$$

start with the initial feasible solutions. All $X_{ijt} = 0$

	Operating Cost (\$)	Emission Cost (\$)
1	488,605,747	73,976,599
2	519,202,084	78,138,516
3	551,204,618	82,027,320
4	588,885,006	83,806,048
5	628,193,291	86,015,684
Total	2,776,090,745	403,964,168
Investment Cost	0	
Total Costs	3,180,054,913	

Relaxed Master 1

Min Z

S.T

$$\sum_Z \sum (C_{ijt} - W_{ijt} \text{Cap}_{ijt}) X_{ijt} + \sum \sum W_{ijt} \text{Cap}_{ijt} X_{ijt} + \sum \sum F_{ijt} \text{EG}(Y_{ijt}) + \sum \sum P_t S_{ijt} \text{EG}(Y_{ijt}) \leq$$

where

$$W_{ijt} = -[F_{ijt} + PS_{ijt}] \partial \text{EG}(Y) / \partial Y_{ijt} - U_t \partial \text{EUE}(Y) / \partial Y_{ijt}$$

X_{ijt} decision variable = 1 if unit i is retrofited with option j at time t is selected
0 else

C_{ijt} the investment cost when unit i is retrofited with option j at time t.

Cap_{ijt} Capacity of unit i with a retrofit j in period t

P_t The price of each allowance assumed = \$300 for all years

S_{ijt} The emission rate of unit i with a retrofit option of j in period j

In order to find W_{ijt} we have first to calculate the derivatives of EG and EUE with respect to Y (which are listed at the end of this appendix)

3	W_{3a}	Cost-CapW	$W-Cap$	W_{3a}	Cost-CapW	$W-Cap$	W_{3a}	Cost-CapW	$W-Cap$
1	-148502	89101292	-89101292	-136008	-81604985	81604985	273595	-109438025	-109438025
2	-150816	90489494	-90489494	-140699	-84419258	84419258	260105	-104041946	-104041946
3	-155868	93520751	-93520751	-147731	-88638685	88638685	248988	-99595002	-99595002
3	-155009	93005301	-93005301	-149108	-89464904	89464904	248988	-95564927	-95564927
5	-159382	95629317	-95629317	-155647	-93388463	93388463	241638	-96655202	-96655202
	W_{3a}			W_{4a}			W_{4a}		
1	-378865	213127258	-213952829	-366386	-208770136	208840136	-598988	239115111	-216535599
2	-153839	84862676	-87688247	-143747	-81865881	81935881	-260295	103638043	-9706239
3	-153459	84646230	-87471801	-145376	-82794140	82864140	-584877	233470899	-210555809
4	-385266	216776249	-219601820	-379434	-216207276	216277276	-247038	98335278	-88933751
5	-165024	91237869	-94063440	-161386	-91920234	91990234	-551222	22008746	-198439871
	W_{3a}			W_{3a}			$W70$	cost-wcap	$W-Cap$
1	-1102112	654574247	-654654247	-238881	-95552575	95552575	-312353	124941062	-124941062
2	-827819	491644458	-491724458	-238625	-95450097	95450097	-300352	120140980	-120140980
3	-153401	91040020	-91120020	-240889	-96355514	96355514	-291064	116425590	-116425590
3	97688	-58106872	58026872	-244332	-97732720	97732720	-275068	110027274	-110027274
5	148939	-88549912	88469912	-260575	-104230143	104230143	-264743	105897066	-105897066
	W_{3a}			W_{3a}					
1	-366386	208641267	-210672067	-1187256	-426912278	427412278	-638356	230258070	-229808070
2	-143747	80623817	-82654617	-238667	-85384244	85884244	-301091	108842684	-108392684
3	-145376	81560219	-83591019	-1216806	-437549997	438049997	-627409	226317308	-225867308
4	-379634	216143645	-218174445	-257436	-92176977	92676977	-283570	102535130	-102085130
5	-161386	90766366	-92797166	-575243	-206587631	207087631	-574618	207312631	-206862631

The relaxed problem is a linear programming problem. We used LINDO software, which is a linear programming optimization software, to solve the relaxed problem.

The output of LINDO is

X_{321} , X_{413} , X_{513} , X_{611} , X_{711} and $X_{ijt} = 0$ for all other ijt .

$$LB = \sum \sum (C_{ijt} - W_{ijt} \text{Cap}_{ijt}) X_{ijt} + \sum \sum W_{ijt} \text{Cap}_{ijt} X_{ijt} + \sum \sum F_{ijt} \text{EG}(Y_{ijt}) + \sum \sum P_t S_{ijt} \text{EG}(Y_{ijt})$$

$$LB = \$2,972,013,937$$

Is UB = LB No

New Variables

X_{321} , X_{413} , X_{513} , X_{611} , X_{711}

Iteration 2

Subproblem

minimize $\sum \sum \text{FEG}(Y) + \sum \sum \text{PSEG}(Y)$

subject to

$$\begin{array}{ll} \text{EUE}(Y) \leq R & \text{for all } t \\ Y \leq \text{Cap } X & \text{for all } ij \text{ and } t \\ Y \geq 0 & \text{for all } ij \text{ and } t \end{array}$$

	Operating Cost (\$)	Emission Cost (\$)
1	504,431,766	52,364,112
2	536,799,498	55,098,211
4	576,005,871	52,856,915
4	624,473,733	39,943,359
5	636,946,003	68,381,584
Costs	2,878,656,871	268,644,181
Total Costs	3,147,301,052	
Investment Costs	1,580,000	
UB	3,149,422,320	

Relaxed Master 2

Min Z

S.T

$$\sum \sum (C_{ijt} - W_{ijt} \text{Cap}_{ijt}) X_{ijt} + \sum \sum W_{ijt} \text{Cap}_{ijt} X_{ijt} + \sum \sum F_{ijt} \text{EG}(Y_{ijt}) + \sum \sum P_t S_{ijt} \text{EG}(Y_{ijt}) \leq Z$$

The output of LINDO is

 X_{325} and $X_{ijt} = 0$ for all other ijt .

$$\text{LB} = \sum \sum (C_{ijt} - W_{ijt} \text{Cap}_{ijt}) X_{ijt} + \sum \sum W_{ijt} \text{Cap}_{ijt} X_{ijt} + \sum \sum F_{ijt} \text{EG}(Y_{ijt}) + \sum \sum P_t S_{ijt} \text{EG}(Y_{ijt})$$

$$\text{LB} = \$2,985,234,427$$

Is UB = LB No

New Variables

 X_{325}

Iteration 3

Subproblem

minimize $\sum \sum \text{FEG}(Y) + \sum \sum \text{PSEG}(Y)$

subject to

$$\begin{aligned} \text{EUE}(Y) &\leq R && \text{for all } t \\ Y &\leq \text{Cap } X && \text{for all } ij \text{ and } t \\ Y &\geq 0 && \text{for all } ij \text{ and } t \end{aligned}$$

	Operating Cost (\$)	Emission Cost (\$)
4	504,431,766	52,364,112
2	536,799,498	55,098,211
3	576,005,871	52,856,915
4	624,473,733	39,943,359
5	636,946,003	68,381,584
Costs	2,878,656,871	268,644,181
Total Costs	3,147,301,052	
Investment Costs	80,000	
UB	3,147,381,052	

The output of LINDO is

$X_{321}, X_{611}, X_{513}, X_{414}, X_{714}$ and $X_{ijt} = 0$ for all other ijt .

$$LB = \sum \sum (C_{ijt} - W_{ijt} \text{Cap}_{ijt}) X_{ijt} + \sum \sum W_{ijt} \text{Cap}_{ijt} X_{ijt} + \sum \sum F_{ijt} EG(Y_{ijt}) + \sum \sum P_t S_{ijt} EG(Y_{ijt})$$

$$LB = \$2,994,190,130$$

Is UB = LB No

New Variables

$X_{321}, X_{611}, X_{513}, X_{414}, X_{714}$ and $X_{ijt} = 0$ for all other ijt .

Iteration 4

Subproblem

minimize $\sum \sum FEG(Y) + \sum \sum PSEG(Y)$

subject to

$$\begin{aligned} EUE(Y) &\leq R && \text{for all } t \\ Y &\leq \text{Cap } X && \text{for all } ij \text{ and } t \\ Y &\geq 0 && \text{for all } ij \text{ and } t \end{aligned}$$

	Operating Cost (\$)	Emission Cost (\$)
4	501,285,647	54,409,089
2	531,846,568	58,317,615
3	569,964,761	56,883,152
4	624,473,733	39,943,359
5	664,877,917	40,990,939
Costs	2,892,448,626	250,544,154
Total Costs	3,142,992,780	
Investment Costs	1,580,000	
UB	3,144,572,780	

The output of LINDO is
 X_{320} and $X_{ijt} = 0$ for all other ijt .

$$LB = \sum \sum (C_{ijt} - W_{ijt} \text{Cap}_{ijt}) X_{ijt} + \sum \sum W_{ijt} \text{Cap}_{ijt} X_{ijt} + \sum \sum F_{ijt} EG(Y_{ijt}) + \sum \sum P_t S_{ijt} EG(Y_{ijt})$$

$$LB = \$3,018,143,651$$

Is $UB = LB$ No

New Variables

X_{320}

Iteration 5

Subproblem

minimize $\sum \sum FEG(Y) + \sum \sum PSEG(Y)$

subject to

$$\begin{aligned} EUE(Y) &\leq R && \text{for all } t \\ Y &\leq \text{Cap } X && \text{for all } ij \text{ and } t \\ Y &\geq 0 && \text{for all } ij \text{ and } t \end{aligned}$$

	Operating Cost (\$)	Emission Cost (\$)
1	496,414,674	58,210,916
2	527,262,336	61,896,506
3	559,164,999	65,270,927
4	597,086,671	66,636,497
5	636,946,003	68,381,584
Costs	2,816,874,683	320,396,430
Total Costs	3,137,271,113	
Investment Costs	80,000	
UB	3,137,351,113	

The output of LINDO is

X_{321} , X_{413} , X_{701} , X_{601} , X_{513} and $X_{ijt} = 0$ for all other ijt .

$$LB = \sum \sum (C_{ijt} - W_{ijt} \text{Cap}_{ijt}) X_{ijt} + \sum \sum W_{ijt} \text{Cap}_{ijt} X_{ijt} + \sum \sum F_{ijt} \text{EG}(Y_{ijt}) + \sum \sum P_t S_{ijt} \text{EG}(Y_{ijt})$$

$$LB = \$3,042,288,801$$

Is $UB = LB$ No

New Variables

X_{321} , X_{413} , X_{701} , X_{601} , X_{513} and $X_{ijt} = 0$ for all other ijt .

Iteration 6

Subproblem

$$\text{minimize } \sum \sum \text{FEG}(Y) + \sum \sum \text{PSEG}(Y)$$

subject to

$$\begin{aligned} \text{EUE}(Y) &\leq R && \text{for all } t \\ Y &\leq \text{Cap } X && \text{for all } ij \text{ and } t \\ Y &\geq 0 && \text{for all } ij \text{ and } t \end{aligned}$$

	Operating Cost (\$)	Emission Cost (\$)
1	496,414,674	58,210,916
2	527,262,336	61,896,506
3	575,345,156	46,838,829
4	613,623,999	47,736,066
5	654,257,514	48,644,915
Costs	2,866,903,680	263,327,232
Total Costs	3,130,230,911	
Investment Costs	650,000	
UB	3,130,880,911	

The output of LINDO is

X_{311} , X_{411} , X_{511} , X_{611} , X_{711} and $X_{ijt} = 0$ for all other ijt .

$$LB = \sum \sum (C_{ijt} - W_{ijt} \text{Cap}_{ijt}) X_{ijt} + \sum \sum W_{ijt} \text{Cap}_{ijt} X_{ijt} + \sum \sum F_{ijt} \text{EG}(Y_{ijt}) + \sum \sum P_t S_{ijt} \text{EG}(Y_{ijt})$$

$$LB = \$3,066,627,111$$

Is $UB = LB$ No

New Variables

X_{311} , X_{411} , X_{511} , X_{611} , X_{711} and $X_{ijt} = 0$ for all other ijt .

Iteration 7

Subproblem

minimize $\sum \sum \text{FEG}(Y) + \sum \sum \text{PSEG}(Y)$

subject to

$$\begin{aligned} \text{EUE}(Y) &\leq R && \text{for all } t \\ Y &\leq \text{Cap } X && \text{for all } ij \text{ and } t \\ Y &\geq 0 && \text{for all } ij \text{ and } t \end{aligned}$$

	Operating Cost	Emission Cost
1	528,176,585	22,549,394
2	562,191,598	23,529,680
3	597,878,706	23,978,334
4	636,476,695	24,668,952
5	676,995,510	25,380,133
Costs	3,001,719,095	120,106,493
Total Costs	3,121,825,587	
Investment Costs	4,325,571	
UB	3,126,151,158	

The output of LINDO is

X_{301} and $X_{ijt} = 0$ for all other ijt .

$$LB = \sum \sum (C_{ijt} - W_{ijt} \text{Cap}_{ijt}) X_{ijt} + \sum \sum W_{ijt} \text{Cap}_{ijt} X_{ijt} + \sum \sum F_{ijt} EG(Y_{ijt}) + \sum \sum P_t S_{ijt} EG(Y_{ijt})$$

$$LB = \$3,078,893,619$$

Is UB = LB No

New Variables

X_{310}

Iteration 8

Subproblem

$$\text{minimize } \sum \sum FEG(Y) + \sum \sum PSEG(Y)$$

subject to

$$\begin{aligned} EUE(Y) &\leq R && \text{for all } t \\ Y &\leq \text{Cap } X && \text{for all } ij \text{ and } t \\ Y &\geq 0 && \text{for all } ij \text{ and } t \end{aligned}$$

	Operating Cost	Emission Cost
1	506,182,742	44,686,027
2	537,405,508	47,973,188
3	570,907,536	50,319,265
3	608,957,885	51,514,283
5	648,868,262	52,976,081
Costs	2,872,321,933	247,468,844
Total Costs	3,119,790,777	
Investment Costs	2,825,571	
UB	3,122,616,348	

The output of LINDO is

X_{311} , X_{411} and $X_{ijt} = 0$ for all other ijt .

$$LB = \sum \sum (C_{ijt} - W_{ijt} \text{Cap}_{ijt}) X_{ijt} + \sum \sum W_{ijt} \text{Cap}_{ijt} X_{ijt} + \sum \sum F_{ijt} \text{EG}(Y_{ijt}) + \sum \sum P_t S_{ijt} \text{EG}(Y_{ijt})$$

$$LB = \$3,104,047,872$$

Is UB = LB No

New Variables

X_{311}, X_{411} and $X_{ijt} = 0$ for all other ijt .

Iteration 9

Subproblem

minimize $\sum \sum \text{FEG}(Y) + \sum \sum \text{PSEG}(Y)$

subject to

$$\begin{array}{ll} \text{EUE}(Y) \leq R & \text{for all } t \\ Y \leq \text{Cap } X & \text{for all } ij \text{ and } t \\ Y \geq 0 & \text{for all } ij \text{ and } t \end{array}$$

	Operating Cost	Emission Cost
1	514,791,561	32,442,326
2	546,536,885	35,041,502
4	581,361,481	36,115,951
4	620,024,059	36,784,859
5	660,414,285	37,639,392
Total	2,923,128,271	178,024,030
Both Costs	3,101,152,301	
Investment Costs	2,895,571	
UB	3,104,047,872	

LB= UB

STOP

The optimal solution is X_{311}, X_{411}

The operating Cost = \$2,923,128,271

The Emission Cost = \$178,024,030

Investment Cost = \$2,895,571

Total Cost = \$3,104,047,872

$$\partial \text{EG}_{i0t} / \partial Y_{30t}$$

i/t	1	2	3	4	5
1	0	0	0	0	0
2	0	0	0	0	0
3	5687	5868	6056	6225	6383
4	-254	-268	-274	-271	-261
5	-75	-107	-134	-157	-174
6	31	-3	-37	-71	-102
7	109	86	58	27	-7
8	73	68	59	47	33
9	74	74	70	63	52
10	69	73	74	72	65
11	59	67	72	74	73
12	110	142	172	198	217
13	-8	12	37	66	96
14	-13	-11	-8	-3	4
15	-14	-13	-11	-8	-3
16	-14	-14	-13	-12	-8
17	-3	-3	-3	-3	-3
18	-3	-3	-3	-3	-3
19	-3	-3	-3	-3	-3
20	-3	-3	16	-3	-3
21	-3	-3	16	-3	-3
22	-3	-3	-3	-3	-3
23	-3	-3	-3	-3	-3
24	-3	-3	-3	-3	-3
25	-2	-3	-3	-3	-3
26	-2	-3	-3	-3	-3
27	-2	-3	-3	-3	-3
28	-2	-3	-3	-3	16
29	-2	-2	-3	-3	16
30	-2	-2	-3	-3	-3
31	-2	-2	-3	-3	-3
32	-2	-2	-3	-3	-3
33	-2	-2	-2	-3	-3
34	-1	-2	-2	-3	-3
35	-1	-2	-2	-3	-3
36	-1	-2	-2	-3	-3

i/t	$\partial EG_{i0t} / \partial Y_{31t}$				
	1	2	3	4	5
1	0	0	0	0	0
2	0	0	0	0	0
3	5730	5910	6097	6264	6421
4	-259	-270	-274	-269	-256
5	-83	-114	-141	-162	-178
6	23	-11	-45	-79	-109
7	104	80	51	19	-15
8	2163	66	57	45	29
9	2165	73	69	61	49
10	70	74	74	2162	64
11	61	68	73	2165	72
12	116	148	177	202	220
13	-5	16	42	71	102
14	-13	-11	-7	-2	5
15	-14	-13	-11	-7	-2
16	-14	-14	-13	-11	-8
17	-3	-3	-3	-3	-3
18	-3	-3	-3	-3	-3
19	-3	-3	-3	-3	36
20	-3	16	-3	-3	36
21	-3	16	-3	-3	-3
22	-3	-3	-3	-3	-3
23	-3	-3	-3	-3	-3
24	-3	-3	-3	-3	-3
25	-3	-3	-3	-3	-3
26	-2	-3	-3	-3	-3
27	-2	-3	-3	-3	-3
28	-2	-3	-3	-3	-3
29	-2	-3	-3	-3	-3
30	-2	-2	-3	-3	16
31	-2	-2	-3	-3	16
32	-2	-2	-3	-3	-3
33	-2	-2	-3	-3	-3
34	-2	-2	-2	-3	-3
35	-1	-2	-2	-3	-3
36	-1	-2	-2	-3	-3

i/t	$\partial EG_{10t} / \partial Y_{32t}$				
	1	2	3	4	5
1	0	0	0	0	0
2	0	0	0	0	0
3	5690	5872	6072	6239	217
4	-255	-269	-274	-8729	-6492
5	-77	5912	-136	-6024	-175
6	5432	6015	-39	-73	-104
7	9986	85	57	25	-9
8	2405	68	59	2540	32
9	74	74	70	2555	51
10	69	73	74	71	65
11	59	67	72	74	72
12	111	143	173	198	218
13	-7	13	38	67	97
14	-13	150	-7	-3	4
15	-14	148	-11	153	-3
16	-14	-14	-13	149	-8
17	-3	-3	-3	-3	-3
18	-3	-3	-3	-3	55
19	-3	-3	-3	-3	55
20	-3	-3	-3	-3	-3
21	-3	-3	-3	-3	-3
22	-3	-3	-3	-3	-3
23	-3	-3	-3	-3	-3
24	-3	-3	-3	-3	-3
25	-2	-3	-3	-3	-3
26	-2	-3	-3	-3	-3
27	-2	-3	-3	-3	-3
28	-2	-3	-3	-3	-3
29	-2	-2	-3	-3	-3
30	-2	-2	-3	-3	-3
31	-2	-2	-3	-3	-3
32	-2	-2	-3	-3	-3
33	-2	-2	-2	-3	16
34	-2	-2	-2	-3	16
35	-1	-2	-2	-3	-3
36	-1	-2	-2	-3	-3

i/t	$\partial \text{EG}_{i0t} / \partial Y_{33t}$				
	1	2	3	4	5
1	0	0	0	0	0
2	0	0	0	0	0
3	5727	5907	6095	6261	6418
4	-258	-270	-274	-269	-257
5	-82	-113	-140	-161	-177
6	25	-9	-44	-78	-108
7	105	4866	52	20	-14
8	72	2560	57	45	30
9	1763	74	69	61	50
10	1759	74	74	71	64
11	60	68	73	74	72
12	115	147	176	3397	220
13	-5	16	1203	3265	101
14	-13	-11	315	-2	5
15	-14	-13	-11	234	-3
16	-14	-14	-13	230	-8
17	-3	-3	-3	-3	-3
18	-3	-3	-3	-3	36
19	-3	-3	-3	-3	36
20	-3	-3	-3	-3	-3
21	-3	-3	-3	-3	-3
22	-3	-3	-3	-3	-3
23	-3	-3	-3	-3	-3
24	-3	-3	-3	-3	-3
25	-2	-3	-3	-3	-3
26	-2	-3	16	-3	-3
27	-2	-3	16	-3	-3
28	-2	-3	-3	-3	-3
29	-2	-2	-3	-3	16
30	-2	-2	-3	-3	16
31	-2	-2	-3	-3	-3
32	-2	-2	-3	-3	-3
33	-2	-2	-3	-3	-3
34	-2	-2	-2	-3	-3
35	-1	-2	-2	-3	-3
36	-1	-2	-2	-3	-3

i/t	$\partial EG_{10t}/\partial Y_{40t}$				
	1	2	3	4	5
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	4339	4573	4808	5043	5275
5	-75	-107	-134	-157	-174
6	31	-3	-37	-71	-102
7	109	86	58	27	-7
8	73	68	59	47	33
9	74	74	70	63	52
10	69	73	74	72	65
11	59	67	72	74	73
12	110	142	172	198	217
13	-8	12	37	66	96
14	-13	-11	-8	-3	4
15	-14	-13	-11	-8	-3
16	-14	-14	-13	-12	-8
17	-3	-3	-3	-3	-3
18	-3	-3	-3	-3	-3
19	-3	-3	-3	-3	-3
20	-3	-3	16	-3	-3
21	-3	-3	16	-3	-3
22	-3	-3	-3	-3	-3
23	-3	-3	-3	-3	-3
24	-3	-3	-3	-3	-3
25	-2	-3	-3	-3	-3
26	-2	-3	-3	-3	-3
27	-2	-3	-3	-3	-3
28	-2	-3	-3	-3	16
29	-2	-2	-3	-3	16
30	-2	-2	-3	-3	-3
31	-2	-2	-3	-3	-3
32	-2	-2	-3	-3	-3
33	-2	-2	-2	-3	-3
34	-1	-2	-2	-3	-3
35	-1	-2	-2	-3	-3
36	-1	-2	-2	-3	-3

i/t	$\partial EG_{i0t}/\partial Y_{41t}$				
	1	2	3	4	5
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	4372	4609	4847	5084	5316
5	-83	-114	-141	-162	-178
6	23	-11	-45	-79	-109
7	104	80	51	19	-15
8	2163	66	57	45	29
9	2165	73	69	61	49
10	70	74	74	2162	64
11	61	68	73	2165	72
12	116	148	177	202	220
13	-5	16	42	71	102
14	-13	-11	-7	-2	5
15	-14	-13	-11	-7	-2
16	-14	-14	-13	-11	-8
17	-3	-3	-3	-3	-3
18	-3	-3	-3	-3	-3
19	-3	-3	-3	-3	36
20	-3	16	-3	-3	36
21	-3	16	-3	-3	-3
22	-3	-3	-3	-3	-3
23	-3	-3	-3	-3	-3
24	-3	-3	-3	-3	-3
25	-3	-3	-3	-3	-3
26	-2	-3	-3	-3	-3
27	-2	-3	-3	-3	-3
28	-2	-3	-3	-3	-3
29	-2	-3	-3	-3	-3
30	-2	-2	-3	-3	16
31	-2	-2	-3	-3	16
32	-2	-2	-3	-3	-3
33	-2	-2	-3	-3	-3
34	-2	-2	-2	-3	-3
35	-1	-2	-2	-3	-3
36	-1	-2	-2	-3	-3

i/t	$\partial EG_{i0t}/\partial Y_{50t}$				
	1	2	3	4	5
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	3961	3956	3959	4092	4369
6	23	-11	-46	-79	-110
7	104	80	51	18	-16
8	73	67	57	44	29
9	75	74	69	61	49
10	71	75	75	71	64
11	62	70	74	75	73
12	121	153	183	207	225
13	-4	18	45	74	106
14	-13	-11	-7	-2	5
15	-15	-14	-11	-8	-2
16	-15	-15	-14	-12	-8
17	-3	-4	-4	-3	-3
18	-3	-4	-4	-3	-3
19	-3	-4	-4	-3	-3
20	-3	-4	16	-4	-3
21	-3	-3	16	-4	-3
22	-3	-3	-4	-4	-3
23	-3	-3	-4	-4	-3
24	-3	-3	-4	-4	-4
25	-3	-3	-3	-4	-4
26	-3	-3	-3	-4	-4
27	-2	-3	-3	-4	-4
28	-2	-3	-3	-4	16
29	-2	-3	-3	-3	16
30	-2	-3	-3	-3	-4
31	-2	-2	-3	-3	-4
32	-2	-2	-3	-3	-4
33	-2	-2	-3	-3	-3
34	-2	-2	-3	-3	-3
35	-2	-2	-2	-3	-3
36	-2	-2	-2	-3	-3

i/t	$\partial EG_{i0t} / \partial Y_{51t}$				
	1	2	3	4	5
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	9330	3912	9490	4147	4427
6	6032	-22	6117	-90	-119
7	97	71	41	8	-26
8	71	64	54	40	2838
9	75	73	67	58	2860
10	73	75	74	70	61
11	1271	71	75	75	71
12	4487	161	2804	213	228
13	1	24	2666	82	114
14	-13	-10	-6	0	7
15	-15	-13	150	-7	-1
16	-15	-15	228	-11	-7
17	-4	-4	16	35	-2
18	-3	-4	-4	35	-3
19	-3	-4	-4	-3	-3
20	-3	-4	-4	16	-3
21	-3	-4	-4	16	-3
22	-3	16	-4	-4	35
23	-3	16	-4	16	35
24	-3	-3	-4	16	-4
25	-3	-3	-4	-4	-4
26	-3	-3	-3	-4	-4
27	-3	-3	-3	-4	-4
28	-2	-3	-3	-4	-4
29	-2	-3	-3	-4	-4
30	-2	-3	-3	-3	-4
31	-2	-3	-3	-3	-4
32	-2	-2	-3	-3	-4
33	-2	-2	-3	-3	-4
34	-2	-2	-3	-3	-4
35	-2	-2	-3	-3	-3
36	-2	-2	-2	-3	-3

i/t	$\partial \text{EG}_{10t} / \partial Y_{60t}$				
	1	2	3	4	5
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	10007	4310	3957	3906	3939
7	8982	69	4757	-4	-35
8	2404	64	2785	33	16
9	76	73	66	53	39
10	1521	76	74	67	57
11	1512	71	76	75	70
12	130	162	3102	222	234
13	1	25	4125	95	126
14	-13	-10	317	2	10
15	-15	-13	-10	-5	2
16	-15	66	-13	-10	236
17	-4	16	-3	-3	56
18	-3	-4	-4	-3	-2
19	-3	-4	-4	-3	-3
20	-3	-4	-4	-3	-3
21	-3	-4	47	-3	-3
22	-3	-3	66	-4	-3
23	-3	-3	65	-4	-3
24	-3	-3	83	16	-3
25	-3	-3	81	-50	-4
26	-3	-3	59	-50	-4
27	-3	-3	76	-49	-4
28	-2	-3	92	-67	-4
29	-2	-3	108	-66	-4
30	-2	-3	124	-64	-4
31	-2	-3	120	-81	-4
32	-2	-2	116	-79	-4
33	-2	-2	112	-66	-4
34	-2	-2	108	-83	16
35	-2	-2	103	-90	609
36	-2	-2	99	-87	650

i/t	$\partial EG_{i0t} / \partial Y_{61t}$				
	1	2	3	4	5
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	4568	4288	4012	3867	3868
7	97	71	41	8	-26
8	71	64	54	40	2838
9	75	73	67	58	2860
10	73	75	74	70	61
11	1271	71	75	75	71
12	4487	161	2804	213	228
13	1	24	2666	82	114
14	-13	-10	-6	0	7
15	-15	-13	150	-7	-1
16	-15	-15	228	-11	-7
17	-4	-4	16	35	-2
18	-3	-4	-4	35	-3
19	-3	-4	-4	-3	-3
20	-3	-4	-4	16	-3
21	-3	-4	-4	16	-3
22	-3	16	-4	-4	35
23	-3	16	-4	16	35
24	-3	-3	-4	16	-4
25	-3	-3	-4	-4	-4
26	-3	-3	-3	-4	-4
27	-3	-3	-3	-4	-4
28	-2	-3	-3	-4	-4
29	-2	-3	-3	-4	-4
30	-2	-3	-3	-3	-4
31	-2	-3	-3	-3	-4
32	-2	-2	-3	-3	-4
33	-2	-2	-3	-3	-4
34	-2	-2	-3	-3	-4
35	-2	-2	-3	-3	-3
36	-2	-2	-2	-3	-3

i/t	$\partial EG_{10t} / \partial Y_{70t}$				
	1	2	3	4	5
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	0	0	0	0	0
7	5510	5228	4952	4659	4380
8	73	67	57	44	29
9	75	74	69	61	49
10	71	75	75	71	64
11	62	70	74	75	73
12	121	153	183	207	225
13	-4	18	45	74	106
14	-13	-11	-7	-2	5
15	-15	-14	-11	-8	-2
16	-15	-15	-14	-12	-8
17	-3	-4	-4	-3	-3
18	-3	-4	-4	-3	-3
19	-3	-4	-4	-3	-3
20	-3	-4	16	-4	-3
21	-3	-3	16	-4	-3
22	-3	-3	-4	-4	-3
23	-3	-3	-4	-4	-3
24	-3	-3	-4	-4	-4
25	-3	-3	-3	-4	-4
26	-3	-3	-3	-4	-4
27	-2	-3	-3	-4	-4
28	-2	-3	-3	-4	16
29	-2	-3	-3	-3	16
30	-2	-3	-3	-3	-4
31	-2	-2	-3	-3	-4
32	-2	-2	-3	-3	-4
33	-2	-2	-3	-3	-3
34	-2	-2	-3	-3	-3
35	-2	-2	-2	-3	-3
36	-2	-2	-2	-3	-3

i/t	$\partial EG_{10t} / \partial Y_{71t}$				
	1	2	3	4	5
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	0	0	0	0	0
7	5462	5178	4901	4608	4330
8	71	64	54	40	2838
9	75	73	67	58	2860
10	73	75	74	70	61
11	1271	71	75	75	71
12	4487	161	2804	213	228
13	1	24	2666	82	114
14	-13	-10	-6	0	7
15	-15	-13	150	-7	-1
16	-15	-15	228	-11	-7
17	-4	-4	16	35	-2
18	-3	-4	-4	35	-3
19	-3	-4	-4	-3	-3
20	-3	-4	-4	16	-3
21	-3	-4	-4	16	-3
22	-3	16	-4	-4	35
23	-3	16	-4	16	35
24	-3	-3	-4	16	-4
25	-3	-3	-4	-4	-4
26	-3	-3	-3	-4	-4
27	-3	-3	-3	-4	-4
28	-2	-3	-3	-4	-4
29	-2	-3	-3	-4	-4
30	-2	-3	-3	-3	-4
31	-2	-3	-3	-3	-4
32	-2	-2	-3	-3	-4
33	-2	-2	-3	-3	-4
34	-2	-2	-3	-3	-4
35	-2	-2	-3	-3	-3
36	-2	-2	-2	-3	-3

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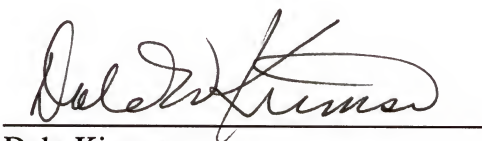
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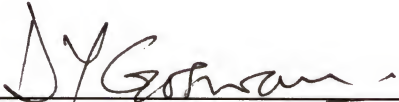
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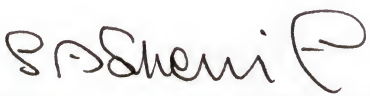
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